

## Mixture Problems

### Amount vs Rate:

Amount is a quantity.

Rate is a ratio that compares two quantities of different units.

If I told you I drove 60 mph for 3 hours and asked how far I had driven, you would not respond 60 mph (a rate) but rather you would state the quantity 180 miles. It is important that you make the distinction between rate and amount, particularly throughout the process of solving a mixture problem. When analyzing mixture problems it is essential to convert to and discuss quantities (**amounts**) not rates.

When analyzing a mixture problem, as in many other problems, a winning strategy is to find some quantity (**amount**) in the problem which can be expressed in two different ways thus arriving at an equation which can be solved.

Keep in mind that one interpretation/application of The Transitive Property of Equations is:

**If two expressions represent the same quantity,  
then those two expressions are equal.**

Study the detailed explanation and analysis of the following mixture problem. In this analysis (an integral part of the solution process) I will state the steps in logical order. Beneath each step, in small blue indented type, I will state the reason/logic/explanation for that step.

**Problem:** What quantity (amount) of a 60% acid solution must be mixed with a 30% acid solution to produce 300 mL of a 50% acid solution?

### Analysis:

(1) Let  $x$  be the **amount** of the 60% solution to be added.

Begin by assigning a variable to the quantity (not the rate) of material you are to determine.

(2) The **amount** of the final solution is 300 mL.

The problem statement dictates that the final quantity be 300 mL.

(3) The **amount** of acid in the **final solution** is  $(0.5)(300)$ .

The problem statement dictates that the final solution be half acid (a 50% solution).

Observe that  $(0.5)(300)$  is one way of describing the amount of acid in the final solution. Our strategy is to find another way of describing the amount of acid in the final solution. Those two ways of describing the amount of acid in the final solution will produce an equation. That equation is the mathematical model for this mixture question. Note that we are **focusing on the amount of acid in the final solution**.

Now try to write the amount of acid in the final solution by calculating the amount contributed by each of the addends

(4) The **amount** of acid contributed by the 60% solution is  $0.6x$

This is the meaning of a 60% solution. If 60% of  $x$  mL is acid then the amount of acid is  $.6x$  mL.

(5) The **amount** of 30% solution is  $300 - x$ .

The sum of the amount of 60% solution and amount of 30% solution is 300. There are  $x$  liters of 60% so there must be  $300 - x$  liters of 30% solution

(6) The **amount** of acid contributed by the 30% solution is  $(0.3)(300 - x)$ .

This is the meaning of a 30% solution. If 30% of  $(300 - x)$  mL is acid then the amount of acid is  $(0.3)(300 - x)$  mL.

(7) The **amount** of acid in the **final solution** is therefore

$$0.6x + (0.3)(300 - x).$$

This is the sum of the amount contributed by the 60% solution (Statement 4) and the amount contributed by the 30% solution (Statement 6).

(8) The model for this mixture question is  $0.6x + (0.3)(300 - x) = (0.5)(300)$

This equation results from Statement 3 and Statement 7.

### **Solve the Equation:**

(9) Ordinary processes are used to solve this equation as illustrated below.

$$0.6x + (0.3)(300 - x) = (0.5)(300) \quad \text{Multiply both sides by 10 to clear decimals.}$$

$$6x + 3(300 - x) = (5)(300) \quad \text{Expand each of the expression}$$

$$6x + 900 - 3x = 1500 \quad \text{Add } -900 \text{ to both sides and add like terms}$$

$$3x = 600 \quad \text{Multiply both sides by } 1/3$$

$$x = 200$$

### **Conclusion:**

(10) 200 mL of 60% acid solution must be added to 100 mL of 30% acid solution to produce 300 mL of 50% acid solution.

**Example 1:** What quantity of a 60% acid solution must be mixed with a 30% solution to produce 300 mL of a 50% solution?

**Analysis:**

Let  $x$  be the amount (measured in milliliters) of 60% solution to be added.

The volume of the final mixture will be 300 mL.

The amount of the 30% solution will be  $(300 - x)$  mL.

The amount of acid in the final solution is  $(0.5)(300) = 150$ .

The amount of acid contributed by the 60% solution is  $(0.6)x$ .

The amount of acid contributed by the 30% solution is  $(0.3)(300 - x)$ .

The amount of acid in the final solution is  $(0.6)x + (0.3)(300 - x)$

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $(0.6)x + (0.3)(300 - x) = 150$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$(0.6)x + (0.3)(300 - x) = 150$$

$$6x + 3(300 - x) = 1500$$

$$3x = 600$$

$$x = 200$$

Therefore we conclude 200 milliliters of 60% solution must be added to 100 milliliters of 30% solution to obtain 300 milliliters of 50% solution.

**Example 2:** A jeweler has five rings, each weighing 18 g. made of an alloy of 10% silver and 90% gold. He decides to melt down the rings and add enough silver to reduce the gold content to 75%. How much silver should he add?

**Analysis:**

Let  $x$  be the amount of silver to be added.

The initial amount of alloy is  $(5)(18) = 90$  g.

The total amount of alloy after the silver has been added is  $(90 + x)$  grams.

The total amount of gold in the final alloy is  $(.75)(90 + x)$ .

The amount of gold in one ring is  $(0.9)(18) = 16.2$  g.

The total amount of gold in the final alloy (the amount in 5 rings) is  $(5)(16.2) = 81$  g.

The amount of gold will not change during this process. **This is an important observation.**

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $(.75)(90 + x) = 81$ .

The mathematical model for this concentration problem is the linear equation in one variable  
 $(0.75)(90 + x) = 81$

Use ordinary means to solve this equation

$$(.75)(90 + x) = 81.$$

$$67.5 + (0.75)x = 81$$

$$0.75x = 13.5$$

$$x = 18$$

Therefore we conclude the jeweler must add 18 grams of silver to obtain an alloy with 75% gold.

**Example 3:** A pot contains 6 L of brine at a concentration of 120 g/L. How much of the water should be boiled off to increase the concentration to 200 g/L?

**Discussion:**

(1) Everything on earth has a density. We measure density in grams per liter or other units of mass per volume.

(2) The amount of the concentrate (solute – that which is dissolved) does not change as a result of boiling off (evaporating) some of the water (solvent – that which does the dissolving).

**Analysis:**

Let  $x$  be the amount (measured in liters) of water boiled off.

The volume of the final mixture will be  $(6-x)L$

The amount of concentrate in the initial mixture is  $(120\text{g/L})(6L) = 720\text{g}$ .

The amount of concentrate in the final mixture is  $(200\text{g/L})(6-x)L = (1200 - 200x)\text{g}$ .

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $(1200 - 200x) = 720$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$1200 - 200x = 720$$

$$-200x = -480$$

$$x = 2.4$$

Therefore we conclude that 2.4 liters of water must be boiled off to obtain a concentration of 200g/L.

**Example 4:** How much water must be added to 20 ounces of a 15% acid solution to reduce it to a 10% acid solution?

**Analysis:**

Let  $x$  be the amount of water to be added.

The amount of the final solution is  $x + 20$ .

The amount of acid in the original solution is  $(0.15)(20) = 3$ .

The amount of acid in the final solution is 3.

The amount of acid in the final solution is  $(0.10)(x + 20) = 0.1x + 2$ .

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $0.1x + 2 = 3$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.1x + 2 = 3$$

$$x + 20 = 30$$

$$x = 10$$

Therefore we conclude that 10 ounces of water must be added to obtain a 10% acid solution.

**Example 5:** How much of a 75% copper alloy should be melted into 62 kg of a 35% copper alloy to produce an alloy which is 50% copper?

**Analysis:** Let  $x$  be the amount of 75% alloy to be added.

The final amount of the alloy will be  $x + 62$ .

The amount of copper in the final alloy will be  $(0.5)(x + 62)$

The amount of copper in the original alloy is  $(0.35)(65) = 22.75$

The amount of copper added is  $0.75x$

The amount of copper in the final alloy is  $0.75x + 22.75$

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $0.75x + 22.75 = (0.5)(x + 62)$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.75x + 22.75 = (0.5)(x + 62).$$

$$75x + 2275 = 50(x + 62)$$

$$75x + 2275 = 50x + 3100$$

$$25x = 825$$

$$x = 33$$

Therefore we conclude that 33 kg of 75% copper alloy should be melted into the 62 kg of 35% copper alloy to produce 95 kg of 50% copper alloy.

**Example 6:** A health clinic uses a solution of bleach to sterilize petri dishes in which cultures are grown. The sterilization tank contains 100 gallons of a solution of 2% ordinary household bleach mixed with pure distilled water. New research indicates that the concentration of bleach should be 5% for complete sterilization. How much of the solution should be drained off and replaced with bleach to increase the bleach content to the recommended level?

**Analysis:**

Let  $x$  be the amount of solution to be drained off (and replaced with bleach).

After the draining has occurred:

The tank contains  $100 - x$  gallons of solution.

The tank contains  $(0.02)(100 - x)$  gallons of bleach.

After the bleach is added:

At the end of the process the tank contains 100 gallons of solution.

Total amount of bleach at the end of the process is  $(0.02)(100 - x) + x$  gallons.

Total amount of bleach at the end of the process is  $(0.05)(100) = 5$  gallons.

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore  $(0.02)(100 - x) + x = 5$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$(0.02)(100 - x) + x = 5$$

$$2 - 0.02x + x = 5$$

$$0.98x = 3$$

$$x = 3.06 \text{ gal.}$$

Therefore we conclude that 3.06 gal. of solution should be replaced with bleach.



**Example 7:** Fred wants to make 100 ml of 5% alcohol solution mixing a quantity of a 2% solution with a 7% alcohol solution. What are the quantities of each of the two solutions he must use?

**Analysis:**

Let  $x$  be the number of milliliters of 7% solution. (call this the strong solution).

Then  $100 - x$  is the number of milliliters of the 2% solution (call this the weak solution).

The total amount of alcohol in the final mixture is 5% of 100 milliliters or  $(0.05)(100) = 5$  milliliters.

The amount of alcohol contributed by the strong solution is 7% of  $x$  or  $0.07x$ .

The amount of alcohol contributed by the weak solution is 2% of  $(100 - x)$  or  $(0.02)(100 - x)$ .

The total amount of alcohol in the final mixture (as contributed by the weak and strong solutions) is  $0.07x + (0.02)(100 - x)$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.07x + (0.02)(100 - x) = 5$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.07x + (0.02)(100 - x) = 5$$

$$7x + 2(100 - x) = 500$$

$$5x = 300$$

$$x = 60$$

Therefore we conclude that 60 milliliters of 7% solution should be mixed with 40 milliliters of 2% solution.

**Example 8:** A chemistry experiment calls for a 30% sulfuric acid solution. If the lab supply room has only 50% and 20% sulfuric acid solutions on hand, how much of each should be mixed to obtain 12 liters of a 30% solution?

**Analysis:**

Let  $x$  be the number of liters of the 50% solution to be used.

Then  $12 - x$  is the number of liters of the 20% solution to be used.

The total amount of sulfuric acid in the final mixture is  $(0.30)(12) = 3.6$

The amount of sulfuric acid contributed by the 50% solution is  $0.5x$ .

The amount of sulfuric acid contributed by the 20% solution is  $(0.2)(12 - x)$ .

The total amount of sulfuric acid in the final mixture is  $0.5x + (0.2)(12 - x)$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.5x + (0.2)(12 - x) = 3.6$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.5x + (0.2)(12 - x) = 3.6$$

$$5x + 2(12 - x) = 36$$

$$3x = 12$$

$$x = 4$$

Therefore we conclude that 4 liters of 50% solution should be mixed with 8 liters of 20% solution to obtain 12 liters of 30% solution.

**Example 9:** How many gallons of a 3% salt solution must be mixed with 50 gallons of a 7% solution to obtain a 5% solution?

**Analysis:**

Let  $x$  be the number of gallons of the 3% solution to be added to the 50 gallons of 7% solution.

The amount of the final mixture will be  $(x + 50)$  gallons.

The final amount of salt in the mixture is 5% of  $(x + 50)$  or  $(.05)(x + 50)$

The amount of salt contributed by the two components of the mixture will be:

3% of  $x$  gallons or  $(.03)x$  and 7% of 50 gallons or  $(.07)(50)$ .

The final amount of salt in the mixture as contributed by the two solutions is  $(.03)x + (.07)(50)$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $(.03)x + (.07)(50) = (.05)(x + 50)$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$(.03)x + (.07)(50) = (.05)(x + 50)$$

$$3x + 7(50) = 5x + 250$$

$$-2x = -100$$

$$x = 50$$

Therefore we conclude that 50 gallons of 3% solution should be mixed with the 50 gallons of 7% solution to obtain a 5% solution.

**Example 10:** To make low fat cottage cheese, milk containing 4% butterfat is mixed with 10 gallons of milk containing 1% butterfat to obtain a mixture containing 2% butterfat. How many gallons of the richer milk is used?

**Analysis:**

Let  $x$  be the number of gallons of 4% milk to be used.

The final mixture will then be  $(10 + x)$  gallons and we want it to contain 2% butterfat.

The total amount of butterfat in the final solution must be 2% of  $(10 + x)$  gallons or  $(.02)(10 + x)$ .

The amount of butterfat contributed by the 10 gallons of 1% milk is  $(.01)(10)$ .

The amount of butterfat contributed by the  $x$  gallons of 4% milk is  $(0.04)(x)$ .

The total amount of butterfat in the final solution is  $(0.1)(10) + (0.04)(x)$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $(.02)(10 + x) = (.01)(10) + (.04)(x)$

This is the model for the question and the answer to the question is obtained by solving the equation.

$$(.02)(10 + x) = (.01)(10) + (.04)(x)$$

$$2(10 + x) = 1(10) + 4x$$

$$20 + 2x = 10 + 4x$$

$$2x = 10$$

$$x = 5$$

Therefore we conclude that 5 gallons of 4% milk should be used to make the cottage cheese.

**Example 11:** Pollution control standards recently changed and now prohibit sale of fuel oil with more than 0.8% sulfur content. The current supply of fuel oil from refineries now has 0.5% sulfur content. An oil distributor has 120,000 gallons of old fuel oil which has 0.9% sulfur content. How many gallons of new fuel oil must the distributor add to the 120,000 gallons to obtain a mixture with 0.8% sulfur content?

**Analysis:**

Let  $x$  be the number of gallons of new oil to be added to the old oil.

The total amount of oil will be  $120,000 + x$ .

The total amount of sulfur in the final mixture will be 0.8% of  $(120,000 + x)$  or  $(0.008)(120000 + x)$ .

The amount of sulfur contributed by the old oil is 0.9% of 120,000 or  $(0.009)(120000) = 1080$ .

The amount of sulfur contributed by the new oil is 0.5% of  $x$  or  $0.005x$ .

The total amount of sulfur in the final mixture will be  $0.005x + 1080$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.005x + 1080 = (0.008)(120000 + x)$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.005x + 1080 = (0.008)(120,000 + x)$$

$$5x + 1,080,000 = 8(120000 + x)$$

$$5x + 1,080,000 = 960,000 + 8x$$

$$1,080,000 - 960,000 = 3x$$

$$120,000 = 3x$$

$$x = 40,000$$

Therefore we conclude that the distributor must add 40,000 gallons of new oil to his 120,000 gallons of old oil.

**Example 12:** A 100% concentrate is to be mixed with a mixture having a concentration of 40% to obtain 55 gallons of a mixture with a concentration of 75%. How much of the 100% concentrate will be needed?

**Analysis:**

Let  $x$  be the amount of the 100% concentrate to be used.

Then  $55 - x$  is the amount of the 40% solution to be used.

The amount of concentrate contributed by the 100% solution is 100% of  $x$  or simply  $x$ .

The amount of concentrate contributed by the 40% solution is 40% of  $(55 - x)$  or simply  $(.4)(55 - x)$ .

The total amount of concentrate is the sum of these two contributions, so the total is  $x + (.4)(55 - x)$ .

However, the total amount of concentrate in the final solution is required to be 75% of 55 gallons or  $(.75)(55)$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $x + (.4)(55 - x) = (.75)(55)$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$100x + (.4)(55 - x) = (.75)(55).$$

$$100x + 40(55 - x) = (75)(55)$$

$$60x = (75)(55) - (40)(55) = (75 - 40)(55) = (35)(55) = 1925$$

$$x = 32.08$$

Therefore we conclude that 32.08 gallons of the 100% solution should be used.

**Example 13:** A chef wants to make 20 ounces of dressing out of vinegar and oil. The vinegar costs \$12.00 per ounce and the oil is \$5.00 per ounce. The dressing should cost \$8.50 per ounce. How much vinegar and how much oil should he use?

**Analysis:**

Let  $x$  be the amount of vinegar.

Then  $20 - x$  is the amount of oil.

The total cost of the mixture is  $20(8.50) = 170$ .

The cost of the vinegar is  $12x$ .

The cost of the oil is  $5(20 - x) = 100 - 5x$ .

The total cost of the mixture is  $12x + 100 - 5x$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $12x + 100 - 5x = 170$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$12x + 100 - 5x = 170$$

$$7x = 70$$

$$x = 10$$

Therefore we conclude that 10 ounces of the vinegar and 10 ounces of oil should be used.

**Example 14:** A grocer mixes peanuts that cost \$2.49 per pound and walnuts that cost \$3.89 per pound to make 100 pounds of a mixture that costs \$3.19 per pound. How much of each kind of nut is put into the mixture?

**Analysis:**

Let  $x$  be the amount of peanuts to be put into the mixture.

Then  $100 - x$  is the amount of walnuts put into the mixture.

The cost of the peanuts in the mixture is  $2.49x$

The cost of the walnuts in the mixture is  $3.89(100 - x)$

The total cost of the final mixture is  $2.49x + 3.89(100 - x)$ .

The total cost of the final mixture is required to be  $(3.19)(100)$  or 319.

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $2.49x + 3.89(100 - x) = 319$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$2.49x + 3.89(100 - x) = 319$$

$$249x + 389(100 - x) = 31900$$

$$249x - 389x = 31900 - 38900$$

$$-140x = -7000$$

$$x = 50$$

Therefore we conclude that the grocer should use 50 pounds of each kind of nut.



**Example 15:** A floral shop creates a mixed arrangement of roses at a cost of \$1.25 each and carnations costing \$0.75 each. An arrangement of 1 dozen flowers costs \$11.00. Determine the number of roses per dozen flowers in the arrangement?

**Analysis:**

Let  $x$  be the number of roses to be used in each arrangement.

Then  $12 - x$  is the number of carnations used in each arrangement.

The cost of the roses in each arrangement is  $1.25x$ .

The cost of the carnations in each arrangement is  $(0.75)(12 - x)$ .

The cost of each arrangement is  $1.25x + (0.75)(12 - x)$ .

The cost of each arrangement is required to be 11.

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $1.25x + (0.75)(12 - x) = 11$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$1.25x + (0.75)(12 - x) = 11$$

$$125x + 75(12 - x) = 1100$$

$$50x = 1100 - (75)(12) = 200$$

$$x = 4$$

Therefore we conclude that the florist should use 4 roses and 8 carnations in each arrangement.

**Example 16:** How many pounds of chocolate worth \$1.20 a pound must be mixed with 10 pounds of chocolate worth 90 cents a pound to produce a mixture worth \$1.00 a pound? Ans 5 lbs.

**Analysis:**

Let  $x$  be the amount of \$1.20 chocolate.

The amount of \$0.90 chocolate is 10.

Then the total amount is  $x + 10$ .

The total cost of the mix is  $1(x + 10) = x + 10$ .

The cost of the expensive chocolate is  $1.2x$ .

The cost of the cheap chocolate is  $10(0.9) = 9$

The total cost of the chocolate as contributed the two types (expensive and cheap) is  $1.2x + 9$

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $1.2x + 9 = x + 10$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$1.2x + 9 = x + 10$$

$$12x + 90 = 10x + 100$$

$$2x = 10$$

$$x = 5$$

Therefore we conclude that 5 pounds of the expensive chocolate must be added to the cheap stuff.

**Example 17:** Soybean meal is 12% protein, cornmeal is 6% protein. How many pounds of each should be mixed together in order to get a 240 lb. mixture that is 9% protein?

**Analysis:**

Let  $x$  be the amount of soybean meal to be used.

Then  $240 - x$  is the amount of cornmeal to be used.

The amount of protein contributed by the soybean meal is 12% of  $x$  or  $0.12x$ .

The amount of protein contributed by the cornmeal is 6% of  $(240 - x)$  or  $(0.06)(240 - x)$ .

The amount of protein in the final mixture is  $0.12x + (0.06)(240 - x)$ .

The amount of protein in the final mixture is required to be 9% of 240 or  $(0.09)(240) = 21.6$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.12x + (0.06)(240 - x) = 21.6$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.12x + (0.06)(240 - x) = 21.6$$

$$12x + 6(240 - x) = 2160$$

$$6x = 2160 - (6)(240) = 720$$

$$x = 120$$

Therefore we conclude that 120 pounds of each of soybean meal and cornmeal should be used.

**Example 18:** Clyde has 2 gallons of gasoline that is 32 parts gasoline and 1 part two-cycle oil. How much gasoline must be added to bring the mixture to 40 parts gasoline and 1 part oil?

**Analysis:**

Let  $x$  be the amount of gasoline to be added.

The total amount of mixture will then be  $x + 2$  gallons.

Each gallon of the original mixture contains  $\frac{1}{33}$  gallon.

The amount of oil in the original 2 gallons of mixture is  $\frac{2}{33}$  gallons. (Amount per gallon times number of gallons)

The amount of oil in the mixture remains constant throughout the mixing process so we may state:

The amount of oil in the final mixture is  $\frac{2}{33}$  gallons.

Each gallon of the final mixture contains  $\frac{1}{41}$  gallon.

The amount of oil in the final mixture is  $\left(\frac{1}{41}\right)(x + 2) = \frac{x + 2}{41}$  (Amount per gallon times number of gallons)

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

$$\text{Therefore } \frac{x + 2}{41} = \frac{2}{33}$$

This is the model for the question and the answer to the question is obtained by solving the equation.

$$\frac{x + 2}{41} = \frac{2}{33}$$

$$x + 2 = \frac{82}{33}$$

$$x = \frac{80}{33} - 2 = \frac{82}{33} - \frac{66}{33} = \frac{16}{33}$$

Therefore we conclude that Clyde should add  $\frac{16}{33}$  gallons (approximately half a gallon) of gasoline to his existing mixture.

**Example 19:** Suppose you buy 100 lbs. of cucumbers that are 99% water. After some time they dehydrate until they are only 98% water. How much do they then weigh?

**Analysis:**

Let  $x$  be the weight of the dehydrated cucumbers.

Observe that the weight of the water in the basket of cucumbers changes as dehydration occurs.

Observe that the weight of the cucumber pulp remains constant throughout the dehydration process.

Therefore we want to focus our attention on the weight of the pulp.

The weight of the cucumber pulp before dehydration is 1% of 100 lbs. or 1 pound.

The weight of the cucumber pulp after dehydration is 2% of  $x$  or  $0.02x$ .

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.02x = 1$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.02x = 1$$

$$2x = 100$$

$$x = 50$$

Therefore we conclude that total weight of the cucumbers after dehydration is 50 lbs.

**Example 20:** Mitch has 352 gallons of gasoline and 15 gallons of oil to make gas/oil mixtures. He wants one mixture to be 9% oil and the other mixture to be 2.5% oil. If he wants to use all of the gas and oil, how many gallons of gas and oil are in each of the resulting mixtures?

**Analysis:**

Let Mix 1 be the mix which contains 9% oil.

Let Mix 2 be the mix which contains 2.5% oil.

Let  $V$  be the volume of Mix 1.

Then the volume of Mix 2 will be  $367 - V$ .

The amount of oil in Mix 1 is 9% of  $V$  or  $0.09V$

The amount of oil in Mix 2 is 2.5% of  $(367 - V)$  or  $(0.025)(367 - V)$

The total amount of oil is the amount of oil in Mix 1 plus the amount of oil in Mix 2.

The total amount of oil in the two mixes is  $0.09V + (0.025)(367 - V)$

Because Mitch wants to use all the oil,

The total amount of oil in the two mixes is 15 gallons.

We have two expressions for the same quantity.

By The Transitive Property of Equality, these two expressions must be equal.

Therefore  $0.09V + (0.025)(367 - V) = 15$ .

This is the model for the question and the answer to the question is obtained by solving the equation.

$$0.09V + (0.025)(367 - V) = 15$$

$$90V + 25(367 - V) = 15000$$

$$65V = 15000 - (25)(367) = 15000 - 9175 = 5825$$

$$V = 89.62 \text{ gallons}$$

Therefore we conclude that:

The volume of Mix 1 is 89.62 gallons.

The volume of Mix 2 is 277.38 gallons.

The amount of oil in Mix 1 is  $(0.09)(89.62) = 8.07$  gallons.

The amount of gasoline in Mix 1 is  $89.62 - 8.07 = 81.55$  gallons.

The amount of oil in Mix 2 is  $(0.025)(277.38) = 6.93$  gallons.

The amount of gasoline in Mix 2 is  $277.38 - 6.93 = 270.45$

The total volume of oil in the two mixes is  $8.07 + 6.93 = 15$  gallons.

The total volume of gasoline in the two mixes is  $81.55 + 270.45 = 352$  gallons