College Algebra **TEST 2 Solution** **Fall 2013**

NAME: ______________________________ Score______________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average__________

No Decimals, mixed numbers, complex fractions, and boxed or circled answers. Use function notation. Show work.

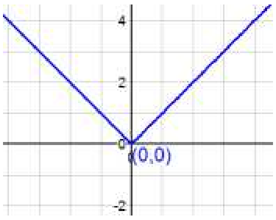
Questions 1 – 20 are 1 pt each

1. **T** F The product of a complex number and its conjugate is its norm.
2. T **F** All linear functions have inverses.
3. T **F** All quadratic functions have inverses.
4. T **F** If both sides of an equation are multiplied by 3x + 2 the resulting equation will be equivalent to the original equation.
5. **T** F A vertical line has no slope.
6. T **F** If the discriminant of a quadratic function is negative and its leading coefficient a is negative then its graph is entirely above the x-axis.
7. **T** F If both sides of an inequality are multiplied by the same negative real number and the inequality symbol is reversed, the resulting inequality is equivalent to the original inequality..
8. T **F** The second coordinate of every point in Quadrant II of the Cartesian Coordinate System is negative.
9. **T** F The domain of a function is a set.
10. T **F** Composition of functions is a commutative operation.
11. The conjugate of 3 – 6i is **3 + 6i**.
12. A quadratic equation in one variable is an equation which can be written in the form $\mathbf{ax^2 + bx + c = 0}$.
13. A quadratic equation in two variables is an equation which can be written in the form $\mathbf{y = ax^2 + bx + c}$.
14. A quadratic function is a function whose rule may be written in the form $\mathbf{f(x) = ax^2 + bx + c}$.
15. The phrase "to factor" means to write as **a product**.
16. The discriminant of a quadratic function $f(x) = ax^2 + bx + c$ is $\mathbf{b^2 - 4ac}$.
17. The composition of a function f with a function g is a function named $f \circ g$ whose rule is $f \circ g(x) = f(g(x))$.
18. The complex component of the complex number 5 – 2i is **– 2** .
19. The graph of a function f is the set of points of the form $(a, f(a))$.
20. Two inequalities are **equivalent** inequalities if they have the same solution sets.

Problems 21 – 32 are 3 points each.

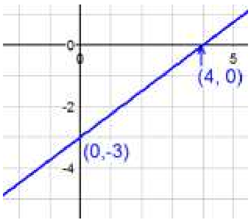
21. Write the rule for the absolute value function and sketch its graph.

abs(x) = |x|
or
f(x) = |x|



22. Sketch the graph of $3x - 4y = 12$. **Show your work. . Label all important points.**

$x = 0 \Rightarrow y = -3 \Rightarrow$ y-intercept is (0,-3)
$y = 0 \Rightarrow x = 4 \Rightarrow$ x-intercept is (4,0)



23. Calculate the norm of 1 + i .

$1^2 + 1^2 = 2$ is the norm

24. Find the multiplicative inverse of the complex number 2 + 3i.

The multiplicative inverse is the conjugate divided by the norm
Therefore the multiplicative inverse of 2 + 3i is $\frac{2-3i}{2^2+3^3} = \frac{2-3i}{13}$

25. Compute the product (2 + i)(3 – 5i) **= 6 – 10i + 3i – 5i^2 = 11 – 7i**

26. Solve the inequality 2x – 5 < 5x + 4. Write the solution set in interval notation.

2x – 5 < 5x + 4

–3x < 9

x > – 9

The solution set is (– 3, ∞)

27. Use the quadratic formula to solve the equation $x^2 + 2x + 3 = 0$ Simplify as much as possible.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 12}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = -1 \pm i\sqrt{2}$$

28. Find the rule for the linear function whose graph has slope 3 and y-intercept 10.

Because the function is linear, its rule is of the form f(x) = mx + b where m is the slope and b is the y-intercept. The desired function has the rule f(x) = 3x – 10.

29. The linear function whose rule is $f(x) = \frac{1}{3}x + 5$ has an inverse. Find the rule for that inverse. Write the rule using function notation.

$$y = \frac{1}{3}x + 5$$
$$x = \frac{1}{3}y + 5$$
$$x - 5 = \frac{1}{3}y$$
$$y = 3x - 15$$
$$f^{-1}(x) = 3x - 15$$

30. Find the rule for the sum of the two functions whose rules are $f(x) = 2x^3 - 5x^2 + 1$ and $g(x) = x^4 - 7$. Do the required additions and write the result in descending order.

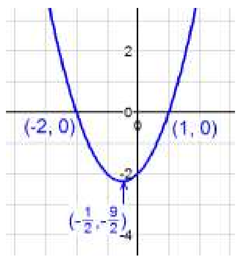
$(f+g)(x) = f(x) + g(x) = (2x^3 - 5x^2 + 1) + (x^4 - 7) = x^4 + 2x^3 - 5x^2 - 6$

Problems 33 – 42 are 5 points each.

31. Sketch the graph of the function whose rule is $f(x) = x^2 + x - 2 = (x + 2)(x - 1)$. **Show your work. Label all important points.**

The graph is a parabola which opens up and has x-intercepts -2 and 1.

The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-1}{2}, f\left(\frac{-1}{22}\right)\right) = \left(-\frac{1}{2}, -\frac{9}{2}\right)$



32. Consider the functions f and g whose rules are $f(x) = 3x + 1$ and $g(x) = x^2 + 1$.
Find the rule for the function $f \circ g$. Simplify as much as possible.

$$f \circ g(x) = f\left(g(x)\right) = f\left(x^2 + 1\right) = 3\left(x^2 + 1\right) + 1 = 3x^2 + 4$$

33. Find the rule for the linear function whose graph passes through the point (5, 4) with slope $\frac{1}{3}$.

Because the desired function is linear it has the form f(x) = mx + b.

Because the desired function has slope $\frac{1}{3}$ **it has the form** $f(x) = \frac{1}{3}x + b$**.**

Because (5, 4) is on the graph f(5) = 4.

But $f(5) = \frac{5}{3} + b$

We have two expressions for the same quantity. By the Transitive Property they must be equal.

Therefore $\frac{5}{3} + b = 4$ and then $b = 4 - \frac{5}{3} = \frac{7}{3}$

It follows that the rule for the desired function is $f(x) = \frac{1}{3}x + \frac{7}{3}$

34. Solve the equation $\sqrt{3x-2}=5$

Square both sides

3x – 2 = 25

3x = 27

x = 9

Test 9: $\sqrt{3(9)-2}=5$ is TRUE

The solution set is {9}.

35. Solve the equation $\frac{x-2}{x+3}=4$

Multiply both sides by x + 3

$x-2=4x+12$

$-3x=14$

$x=-\frac{14}{3}$

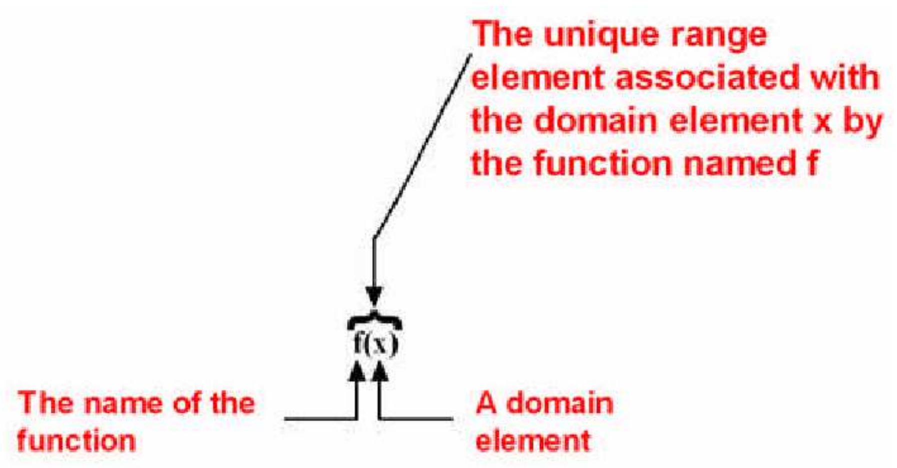
$-\frac{14}{3}$ does not make any denominator 0 so it is a solution.

The solution set is $\left\{-\frac{14}{3}\right\}$

36. Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.

37.



38. The orientation of the graph of a quadratic function whose rule is $f(x) = ax^2 + bx + c$ is controlled by the leading coefficient a and the discriminant $b^2 - 4ac$.

Associate with each graph the correct statement about the leading coefficient and the discriminant.

Fill in the top red blank with the correct one of: $\mathbf{a < 0}$ $\mathbf{a = 0}$ $\mathbf{a > 0}$

Fill in the bottom red blank with the correct of: $\mathbf{b^2 - 4ac < 0}$ $\mathbf{b^2 - 4ac = 0}$ $\mathbf{b^2 - 4ac > 0}$

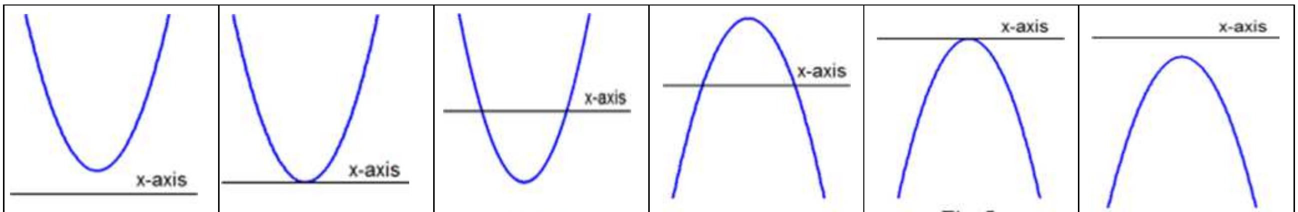
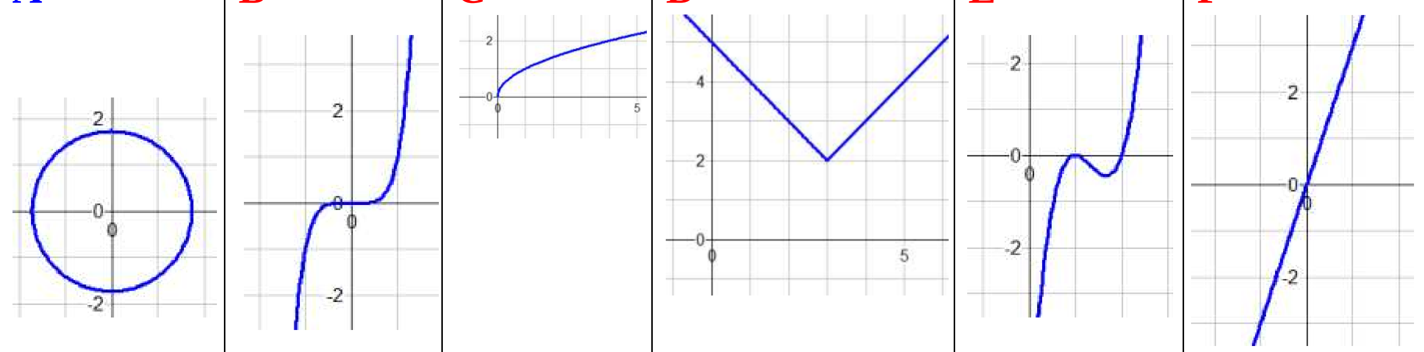


Fig. 1	Fig. 2	Fig. 3	Fig. 4	Fig. 5	Fig. 6
$a > 0$	$a > 0$	$a > 0$	$a < 0$	$a < 0$	$a < 0$
$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$

39. For each of the six graphs below circle the identifying letter if the graph is the graph of a function.

A **B** **C** **D** **E** **F**



40. **(** For each of the six graphs below circle the identifying letter if the corresponding function has an inverse.

A **B** **C** **D** **E** **F**