College Algebra **TEST 3** **Fall 2013**

NAME: ______________________________Score____________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average__________

No Decimals, mixed numbers, complex fractions, and boxed or circled answers. Use function notation. Show work.

Questions 1 – 20 are 2 pts. each

1. **T** F The graph of an odd degree polynomial function must have at least one x-intercept.
2. T **F** The graph of an even degree polynomial function must have at least one x-intercept .
3. T **F** The graph of the equation $y^2 = 16 - x^2$ is a parabola.
4. T **F** If k is a real zero of a polynomial function f and it has odd multiplicity, then the graph of f intersects but does not cross the x-axis.
5. **T** F If f is a polynomial function such f(a) < 0 and f(b) > 0, then the graph of f crosses the x-axis between a and b.
6. T **F** The graph of a function f is the set of points of the form (f(x), x).
7. **T** F Every term is a polynomial.
8. T **F** Every polynomial is a term.
9. T **F** Every quadratic polynomial contains three terms.
10. **T** F If a 7^{th} degree polynomial is divided by a 4^{th} degree polynomial, the quotient will be a 3^{rd} degree polynomial.

11. If $\frac{p}{q}$ is a rational zero of a polynomial function f then **q** is a divisor of the leading coefficient.

12. If $\frac{p}{q}$ is a rational zero of a polynomial function f then **p** is a divisor of the constant term.

13. A circle is the set of points in a plane that are **equidistant** from a fixed point called the center.

14. The numerical part of a term is called the **coefficient** of the term.

15. A **polynomial** is a term or a sum of terms in which all variables have whole number exponents.

16. The product of two terms is the term obtained by multiplying the coefficients according to normal arithmetic properties and multiplying the variables according to the laws of **exponents**.

17. Two polynomials are equal if they have the same degree and corresponding **coefficients** are equal.

18. A third degree polynomial is called a **cubic** polynomial.

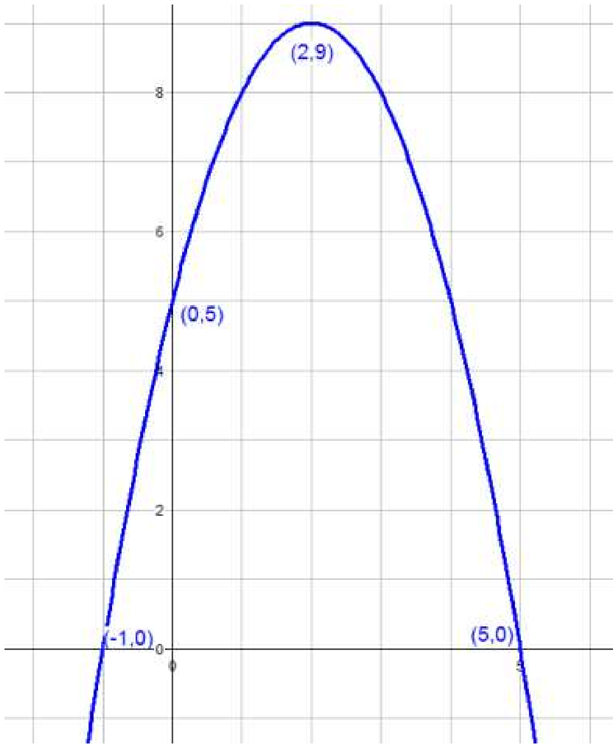
19. A polynomial consisting of two **terms** is called a binomial.

20. Complex zeros of polynomial functions occur in **conjugate** pairs.

21. **(3 pts.)** Sketch the graph of the function whose rule is $f(x) = -(x-5)(x+1)$. **Label all important points.**

f is a quadratic function with negative leading coefficient so its graph is a parabola which opens down.

From the factorization it is clear that the x-intercepts are (-1, 0) and (5, 0)



22. **(3 pts.)** Sketch the graph of the function whose rule is $f(x) = (x-1)(x+4)^2(x-5)^3$. **Label all important points.**

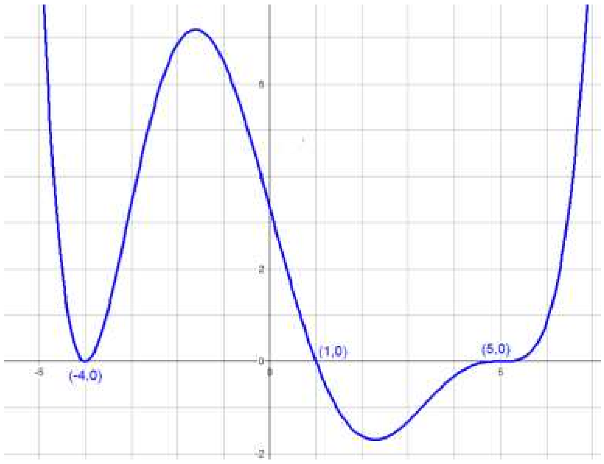
The leading term is x^6.

Therefore $\text{As } x \to +\infty, f(x) \to +\infty$
$\text{As } x \to -\infty, f(x) \to +\infty$

1 is a zero of multiplicity 1 (odd) so the graph crosses the x-axis at (1,0).

- 4 is a zero of multiplicity 2 (even) so the graph intersects but does not cross the x-axis at (-4, 0).

5 is a zero of multiplicity 3 (odd) so the graph crosses the x-axis at (5, 0). Moreover, because the multiplicity is greater than 1, the graph kinda flattens out at (5, 0)



23. **(3 pts.)** Solve $(x-1)(x+4)^2(x-5)^3 < 0$.**Hint: Use #22.**
Write the solution set using interval notation.

The solution set is the interval (1, 5)

24. **(6 pts.)** Consider the graph in Fig. 1. It is the graph of a polynomial function named f. Use interval notation to answer the following three questions.

a. Where is f(x) = 0?

for $x \in \{-1, 2, 4, 5\}$

b. Where is f(x) > 0?

for $x \in (-1, 2) \cup (2, 4) \cup (5, \infty)$

c. Where is f(x) < 0?

for $x \in (-\infty, -1) \cup (4, 5)$

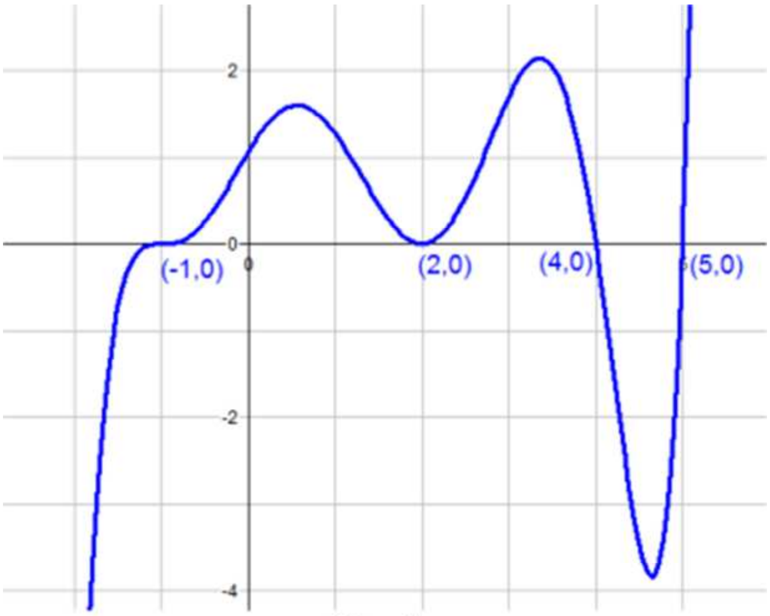


Fig. 1

25. **(3 pts.)** Suppose $\frac{p}{q}$ is a rational zero of a polynomial function with integer coefficients.

Suppose that $p \in \{\pm1, \pm2, \pm5\}$ and $q \in \{\pm1, \pm3\}$.

What are the possible rational zeros?

$\frac{p}{q} \in \left\{\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm5, \pm\frac{5}{3}\right\}$

26. **(3 pts.)** Consider the polynomial function whose rule is $f(x) = 7x^4 + 3x^3 + 14x^2 - 9$.

If $\frac{p}{q}$ is a rational zero of f, then $p \in \{$**±1, ±3, ±9**$\}$

27. **(3 pts.)** Consider the polynomial function whose rule is $f(x) = 4x^7 + 2x^5 + 14x - 5$.

If $\frac{p}{q}$ is a rational zero of f, then $q \in \{$**±1, ±2, ±4**$\}$

28. **(2 pts.)** Write the equation of the circle with center (–5, 2) and radius 7.

$(x + 5)^2 + (y - 2)^2 = 7^2$

29. **(3 pts.)** The graph of the equation $x^2 + y^2 + 4x - 8y = 16$ is a circle. Write its equation in standard form.

$(x^2 + 4x \quad) + (y^2 - 8y \quad) = 16$
$(x^2 + 4x + 4) + (y^2 - 8y + 16) = 16 + 4 + 16 = 36$
$(x + 4)^2 + (y - 4)^2 = 6^2$

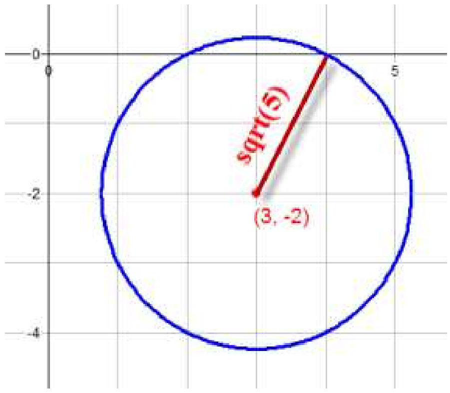
30. **(3 pts.)** The graph of the equation $x^2 + y^2 + 6x + 10y - 2 = 0$ is a circle. What is its center and what is its radius?

$(x^2 + 6x \quad) + (y^2 + 10y \quad) = 2$
$(x^2 + 6x + 9) + (y^2 + 10y + 25 \quad) = 2 + 9 + 25 = 36$
$(x + 3)^2 + (y + 5)^2 = 6^2$
The center is (-3, -5)
The radius is 6.

31. **(3 pts.)** What must be added to $x^2 + 3x$ to create a perfect square?

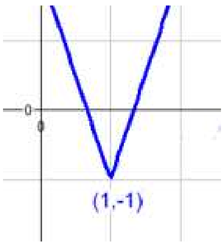
$$\left(\frac{3}{2}\right)^2$$

32. **(2 pts.)** Sketch the graph of the equation $(x - 3)^2 + (y + 2)^2 = 5$.
Draw a radius and label it with its length. Label the center with its coordinates.

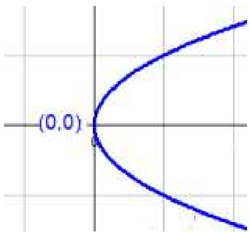


33. **(5 pts.)** Which of the following cannot be the graph of a polynomial function. Circle the letter of those which cannot be graphs of polynomials.

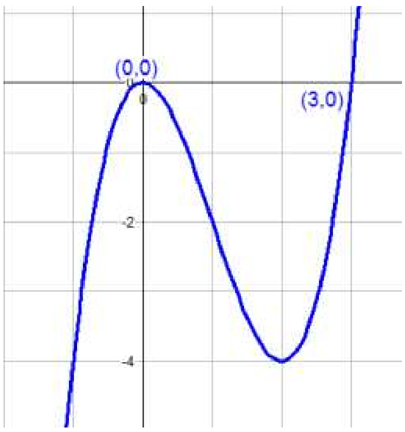
a.



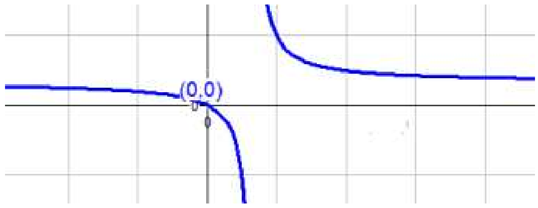
c.



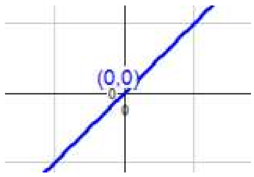
b.



d.



e.



34. **(8 pts.)** Consider the polynomial function whose rule is $f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$. Its graph is shown in Fig. 2.

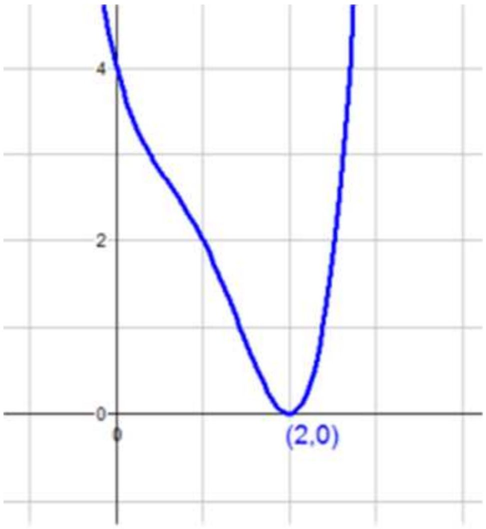


Fig. 2

a. What is a probable rational zero? **2**

A rational zero is a rational number k such that f(k) = 0.
(2, 0) is not a number. So it cannot be a rational zero.

b. What do you expect its multiplicity to be? **2**

Because the curve seems to intersect but not cross the x-axis at (2, 0) I suspect the multiplicity to be even. The only possibilities then are 2 and 4. However, if the multiplicity were 4, I suspect the curve would b "flatter" around (2, 0). Therefore I expect the multiplicity to be 2.

c. Verify that your answer to Part a is indeed a rational zero.

$f(2) = 2^4 - 4(2)^3 + 5(2)^2 - 4(2) + 4 = 16 - 32 + 20 - 8 + 4 = 0$
Therefore by the definition of zero of a function, 2 is a zero of f.

d. Find all the zeros of the function f. **That means all rational, irrational, and complex zeros.**

Because I expect 2 to be a zero of multiplicity 2, I expect $(x - 2)^2 = x^2 - 4x + 4$ to be a factor of $x^4 - 4x^3 + 5x^2 - 4x + 4$. So do the following division.

$$
\begin{array}{r}
x^2 + 1 \\
x - 2\overline{)\,x^4 - 4x^3 + 5x^2 - 4x + 4} \\
\underline{x^4 - 4x^3 + 4x^2} \\
x^2 - 4x + 4 \\
\underline{x^2 - 4x + 4} \\
0
\end{array}
$$

Therefore $f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4 = (x - 2)^2(x^2 + 1)$
Zeros of f are solutions of the equation $(x - 2)^2(x^2 + 1) = 0$
By The Zero Factor property this is equivalent to
$(x - 2)^2 = 0$ OR $(x^2 + 1) = 0$
$x = 2$ OR $x^2 = -1$
$x = 2$ OR $x = \pm i$
The zeros of f are 2, i, and – i .

35. **(3 pts.)** Which of the following are terms?

a. –5
b. x – 5
c. 3x
d. x + y
e. 3 + w
f. 3xtw

36. **(5 pts.)** Which of the following are the rules for polynomial functions

a. $f(x) = x + 5$
b. $f(x) = x^5$
c. $f(x) = 3x^2 - 5x + 12x^{-1}$
d. $f(x) = \frac{3}{4}x^4 - \frac{2}{3}x^3 + \sqrt{2}x - \frac{8}{\sqrt{7}}$
e. $f(x) = \frac{x^2 + 2x - 5}{x}$
f. $f(x) = x^7 - 5$
g. $f(x) = (x - 2)^7(x + 1)$
h. $f(x) = \frac{x+1}{x-1}$
i. $f(x) = \frac{x^2 + 3x - 4}{5}$
j. $f(x) = \frac{5}{x^2 + 3x - 4}$

37. **(2 pts.)** If f is a polynomial function whose rule is $f(x) = -3x^8 + 7x^7 - 2x + 5$, then
As $x \to +\infty, f(x) \to -\infty$
As $x \to -\infty, f(x) \to -\infty$