No Decimals No mixed numbers No complex fractions No boxed or circled answers

College Algebra   TEST 4 Solution   Fall 2013

NAME: ______________________________ Score______________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION   Course Average___________

Questions 1 – 30 are 2 pt each; others are 4 pts each.

1. **T** F The sum of two matrices with different orders is not defined.
2. **T** F The sum of a matrix and its opposite is the zero matrix.
3. T **F** The inner (dot) product of two matrices is a matrix.
4. T **F** Matrix multiplication is commutative.
5. **T** F Matrix addition is commutative.
6. **T** F Matrix multiplication is associative.
7. T **F** Each square matrix has an inverse.
8. **T** F The product (if it is defined) of two matrices is a matrix .
9. T **F** Some non-square matrices have inverses.
10. **T** F The entry in the 23 position of a product matrix is the inner (dot) product of the 2^{nd} row of the first matrix and the 3^{rd} column of the second matrix.
11. T **F** Adjoining the identity matrix to a matrix A is an elementary row operation.
12. T **F** The product of a scalar and a matrix is a number.
13. Two systems of equations are **equivalent** systems if they have the same solution sets.
14. In a system of linear equations, if the value of one of the variables is known, an equivalent system is generated if that value is **substituted** into the equations.
15. A **system** of equations consists of two or more equations involving the same variables.
16. The solution of a matrix equation is a **matrix**.
17. A solution for a system of equations in two variables is an ordered **pair** of numbers which make all of the equations in the system of equations true.
18. Two matrices are row **equivalent** if one can be obtained from the other by a sequence of elementary row operations.
19. Two matrices are **equal** if they have the same order and their corresponding entries are equal.
20. The **identity** matrix of order nXn is the matrix whose main diagonal entries are 1 and all other entries are 0.
21. The product of two matrices is defined only if the number of **columns** of the first is equal to the number of **rows** of the second.
22. If a matrix has m rows and n columns, the **order** of the matrix is $m \times n$.
23. A **matrix** is a rectangular array of numbers.
24. Two square matrices A and B of the same order are **inverses** of each other if AB = I and BA = I where I is the identity matrix of the same order as A and B.
25. **Interchanging** two rows is an elementary row operation.
26. If A and B are matrices and A has order 4X3 while B has order 3X5, then the order of AB is **4 X 5**

No Decimals   No mixed numbers   No complex fractions   No boxed or circled answers

27. **Multiplying** a row of a matrix by a non-zero constant and replacing that row with the product is an elementary row operation.

28. **Adding** a multiple of a row to another row and replacing one but not both of the rows with that sum is an elementary row operation.

29. The solution of a system of two equations in two variables is an ordered **pair** of numbers.

30. The solution of a system of three equations in three variables is an ordered **triple** of numbers.

31. Compute the inner product: $\begin{bmatrix} 2 & -3 & \frac{2}{3} \end{bmatrix} \bullet \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} = \mathbf{4}$

32. Consider the matrices. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ $X = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and

$$\boldsymbol{A}^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

Solve the matrix equation AX = C. Your answer should be a matrix.

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 29 \\ -25 \\ 10 \end{bmatrix}$$

33. Supply the missing entries by performing the indicated elementary row operation.

$$\begin{bmatrix} 1 & 2 & \frac{1}{5} & 0 \\ 5 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{-5R_1 + R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{1} & \boxed{2} & \boxed{\frac{1}{5}} & \boxed{0} \\ \boxed{0} & \boxed{-13} & \boxed{-1} & \boxed{1} \end{bmatrix}$$

No Decimals No mixed numbers No complex fractions No boxed or circled answers

34. Solve the system of equations with the **SUBSTITUTION** method. **(No credit for any other method)**

$$\begin{cases} 2x+3y=1 \\ -3x+y=-2 \end{cases} \longrightarrow \begin{cases} 2x+3y=1 \\ y=3x-2 \end{cases} \longrightarrow \begin{cases} 2x+3(3x-2)=1 \\ y=3x-2 \end{cases}$$

$$\longrightarrow \begin{cases} 11x-6=1 \\ y=3x-2 \end{cases} \longrightarrow \begin{cases} x=\frac{7}{11} \\ y=3x-2 \end{cases} \longrightarrow \begin{cases} x=\frac{7}{11} \\ y=-\frac{1}{11} \end{cases}$$

35. Write the coefficient matrix of the following system of equations: $\begin{cases} -2x+2y-4z=1 \\ 2x-5y-z=6 \\ 4x+2y-3z=5 \end{cases}$

$$\begin{bmatrix} -2 & 2 & -4 \\ 2 & -5 & -1 \\ 4 & 2 & -3 \end{bmatrix}$$

36. Write a matrix equation which is equivalent to the system of equations: $\begin{cases} -2x+2y-4z=1 \\ 2x-5y-z=6 \\ 4x+2y-3z=5 \end{cases}$

$$\begin{bmatrix} -2 & 2 & -4 \\ 2 & -5 & -1 \\ 4 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}$$

37. Perform the multiplication: $\begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 2 & 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -1 & 2 \end{bmatrix}=\begin{bmatrix} 9 & -2 \\ -5 & 6 \\ 11 & 3 \end{bmatrix}$

38. The inverse of $A=\begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$ **is the matrix** $A^{-1}=\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Use this information to solve the system $\begin{cases} -2x - 3y + z = 2 \\ -3x - 3y + z = 0 \\ -2x - 4y + z = 3 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

The solution for the system is the ordered triple (2, -1, 3).

39. Show that the matrices $\begin{bmatrix} -2 & 1 & -1 \\ -5 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ -1 & 2 & 1 \end{bmatrix}$ are not inverses.

$$\begin{bmatrix} -2 & 1 & -1 \\ -5 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Because the product is not the inverse matrix, the two matrices are not inverses.

No Decimals No mixed numbers No complex fractions No boxed or circled answers

40. You do not need to do any computations. Simply fill in the blanks to describe the process for finding the inverse of a matrix.

To find the inverse of the matrix $A = \begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix}$

Begin by adjoining the **identity** matrix to obtain the matrix $\begin{bmatrix} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{bmatrix}$ with order 3×6

The next step is to get a **1** in the **11** position.

Then use that **1** to get **0** everywhere else in the **first column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 7/5 & 4/5 & 1/5 & 0 & 0 \\ 0 & -26/5 & 3/5 & -3/5 & 1 & 0 \\ 0 & -7/5 & 1/5 & -6/5 & 0 & 1 \end{bmatrix}$

The next step is to get a **1** in the **22** position.

Then use that **1** to get **0** everywhere else in the **second column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 25/26 & 1/26 & 7/26 & 0 \\ 0 & 1 & -3/26 & 3/26 & -5/26 & 0 \\ 0 & 0 & 1/26 & -27/26 & -7/26 & 1 \end{bmatrix}$

The next step is to get a **1** in the **33** position.

Then use that **1** to get **0** everywhere else in the **third column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{bmatrix}$

The inverse of A is the matrix $A^{-1} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -7 & 26 \end{bmatrix}$

Which has order 3×3