College Algebra **TEST 3** **Solution** **Fall 2014**

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average_____

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 16 are each worth one point.

1. **T** F A graph which fails the vertical line test is not the graph of a function.
2. **T** F A function f has an inverse if and only if its graph passes the horizontal line test.
3. **T** F A point is above the x-axis if and only if its second coordinate is positive.
4. T **F** A function f has an inverse if and only if its graph passes the vertical line test.
5. **T** F A circle is the set of points in a plane that are equidistant from a fixed point called the center.
6. **T** F All non-constant linear functions have inverses.
7. **T** F The distance formula is an equation.
8. T **F** $x^2 + y^2 = 2$ is the equation of the circle whose center is at the origin and whose radius is 2.
9. T **F** If the rule for a function f is f(x) = 5x – 3, then (7, 2) is on the graph of f.
10. T **F** T he graph of the square root function is a parabola which opens up.
11. T **F** If f and g are inverse functions then $f \circ g(x) = 1$.
12. T **F** The graph of a quadratic function is a parabola which opens up if a < 0 and opens down if a > 0.
13. T **F** The graph of the zero function is the y-axis.
14. T **F** Every function has an inverse.
15. T **F** If the discriminant of a quadratic function is negative, its graph has exactly one x-intercept.
16. **T** F Composition of functions is not commutative.

Questions 17 – 28 are each worth 2 points.

17. State the distance formula – the distance between two points.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

18. Write the equation of the circle with center (4, –7) and radius 2.

$$(x-4)^2 + (y+7)^2 = 4$$

19. Write the standard form for the equation of a circle with center (t, p) and radius k.

$$(x-t)^2 + (y-p)^2 = k^2$$

20. The composition of a function f with a function g is a function named $f \circ g$ whose rule is

$$f \circ g(x) = f(g(x))$$

21. The graph of a function is the set of all points of the form **(a,f(a))** where a is an element of the domain and f(a) is the corresponding range element.

22. Use function notation to write the rule for the identity function. **I(x) = x**

23. Use function notation to write the rule for the squaring function **sq(x) = x^2**.

24. The x-intercept of $3x + 8y = 15$ is **5**.

25. The graph in Fig. 1 is the graph of $f(x) = ax^2 + bx + c$. From the graph you can determine:

    a. $a > 0$

    b. $b^2 - 4ac < 0$

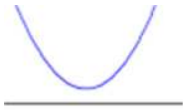


Fig. 1

26. If $f(x) = \dfrac{\sqrt{2x-1}}{x^2}$ then f(2) = $\dfrac{\sqrt{3}}{4}$

27. Write the slope-intercept form of the equation of a non-vertical line. **$y = mx + b$**

28. To verify (prove) that two functions f and g are inverses of each other it is necessary to show that both of the following are true:

$$f \circ g(x) = x$$
$$g \circ f(x) = x$$

Questions 29 – 38 are each worth 5 points.

29. Find the inverse of the function whose rule is $f(x) = 3x + 5$

$$f(x) = 3x + 5$$
$$y = 3x + 5$$
$$x = 3y + 5$$
$$y = \frac{x-5}{3}$$
$$f^{-1}(x) = \frac{x-5}{3}$$

30. Calculate the distance between (3, –2) and (–5,7).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3+5)^2 + (-2-7)^2} = \sqrt{64+81} = \sqrt{145}$$

31. Find the midpoint of the line segment joining (3,2) and (5,1).

$$\text{The midpoint is } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3+5}{2}, \frac{2+1}{2}\right) = \left(4, \frac{3}{2}\right)$$

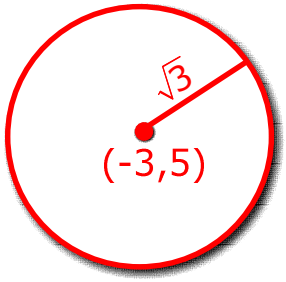
32. Find the slope of the line through (5, –3) and (8, 5) .

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5-(-3)}{8-5} = \frac{8}{3}$$

33. Calculate the discriminant of $2x^2 - 4x + 1$.

$$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$$

34. Sketch the graph of $(x + 3)^2 + (y - 5)^2 = 3$



Use completing the square to write $x^2 + y^2 + 2x + 12y - 12 = 0$ in standard form.

$$x^2 + y^2 + 2x + 12y - 12 = 0$$
$$(x^2 + 2x\ \ ) + (y^2\ + 12y\ \ ) = 12$$
$$(x^2 + 2x + 1) + (y^2\ + 12y + 36) = 12 + 1 + 36 = 49$$
$$(x + 1)^2 + (y + 6)^2 = 7^2$$

35. Find the rule for the linear function whose graph passes through (2,5) with slope 3. Do NOT use the point slope equation. Use function notation.
Because the desired function is linear it has the form f(x) = mx + b.
Because its slope is 3, it has the form f(x) = 3x + b **(*)**
Because (2,5) is on the graph f(2) = 5
From the rule **(*)** f(2) = 6 + b
We have two expressions equal to the same quantity so from the transitive property they must be equal.
Therefore 5 = 6 + b and clearly b = – 1.
The rule for the desired function is f(x) = 3x – 1.

36. Find two functions f and g whose composition $f \circ g$ is $h(x) = \left(2x^3 + x - 4\right)^3$.

Let $g(x) = 2x^3 + x - 4$
Let $f(x) = x^3$

Then $f \circ g(x) = f\left(g(x)\right) = \left(2x^3 + x - 4\right)^3 = h(x)$

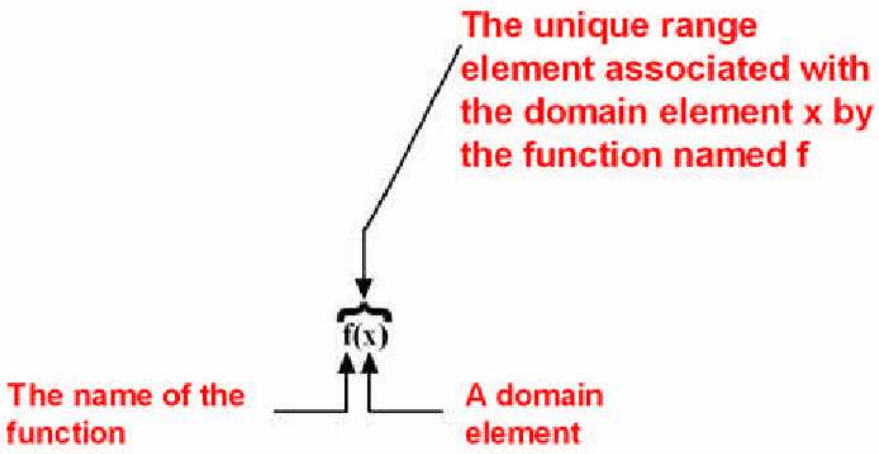
37. Show that f(x) = 2x – 5 and g(x) = x^2 are not inverses.

$f \circ g(x) = f\left(g(x)\right) = f\left(x^2\right) = 2(x^2) - 5 = 2x^2 - 5 \neq x$

OR

$g \circ f(x) = g\left(f(x)\right) = g(2x - 5) = (2x - 5)^2 \neq x$

38. (4 points).



39.(6 pts)Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.