College Algebra **TEST 1 Solution** **Spring 2013**

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average_____

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 30 are 1 pt each

1. T F A formula must be an equation.
2. T F Definitions in mathematics are stipulative.
3. T F A binary operation is a comparison of two operands.
4. T F Deductive reasoning is an important tool in mathematics.
5. T F Two equations are equivalent if they have the same solution sets.
6. T F A real number which is not irrational is rational.
7. T F If 3x + 5 is added to both sides of an equation, the new equation is equivalent to the original equation.
8. T F To say 3 is a solution of an equation means the same as saying {3} is the solution set for that equation.
9. T F An equation is a mathematical statement which contains an = symbol.
10. T F When both sides of an equation are multiplied by an expression containing a variable the resulting equation will be equivalent to the original.
11. The complex component of the complex number 3 – 7i is – 7 .
12. The multiplicative inverse of a complex number a + bi is its conjugate divided by its norm.
13. A linear equation in one variable is an equation which can be written in the form ax + b = 0.
14. The set of Integers consists of the set of Whole numbers with their opposites adjoined.
15. A number that makes an equation true when substituted for the variable is a solution of the equation.
16. The graph of an equation in one variable is drawn on the Real Number Line.
17. The graph of an equation consists of all the points, and only those points, whose coordinates are solutions of the equation.
18. A conditional inequality is an inequality which is true when some real numbers are substituted for the variables and is false when some real numbers are substituted for the variables.
19. Write a Whole number which is not a Natural number. 0
20. The solution set for a linear identity in one variable is **R**.
21. To solve an equation of the form mx = b, one should multiply both sides of the equation by the multiplicative inverse of m.
22. The process to solve a linear equation in one variable is to generate a sequence of equations each equivalent to the previous equation until a simplest equation is obtained.

23. The discriminant of a quadratic $3x^2 + 5x - 1$ is $5^2 - (4)(3)(-1) = 25 + 12 = 37$.

24. Write the formula for the area of a trapezoid $A = \frac{1}{2}(B + b)h$.

25. Write the formula for the area of a circle $A = \pi r^2$.

26. Two complex numbers a + bi and c + di are equal if $a = c$ and $b = d$.

27. Complete the statement of the quadratic formula: The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

28. When squaring both sides of an equation is part of the solution process, testing all the possible solutions in the original equation must also be a part of the solution process.

29. When both sides of an equation are squared the solution set of the resulting equation contains the solution set of the original equation.

30. It two expressions represent the same quantity, the two expressions must be equal.

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices.
For Questions 31 – 36 each part is worth one point

31. Which of the following are linear equations in one variable
    a. $3x - 7 = 2x + 12$
    b. $\sqrt{7}x + 1 = 0$
    c. $x^2 = 3$
    d. $\frac{2x}{5} + \sqrt{3} = \frac{3}{\sqrt{2}}x$
    e. $\sqrt{x} = 5$

32. The solution set of a conditional linear equation in one variable may be:
    a. The empty set
    b. A set containing one number
    c. A set containing two numbers
    d. We must solve the equation to figure out how many solutions there are
    e. All real numbers $\mathbf{R}$

33. The graph of a conditional linear equation in one variable
    a. Is on the number line
    b. Is in the Cartesian coordinate system
    c. Is a line
    d. Is a dot
    e. Is a collection of more than one dot
    f. Is an interval
    g. Is a ray

34. If the solution set for an equation is the empty set, then
    a. The equation is an identity
    b. The equation is a conditional equation
    c. The equation is a contradiction
    d. The equation has no solution
    e. Every real number is a solution.

35. If both sides of an equation are multiplied by a non-zero real number
    a. the resulting equation is equivalent to the original equation.
    b. the two equations have the same solution sets.
    c. the two equations might have different solution sets.
    d. the two equations are equal.
    e. the resulting equation is a simplest equation.

36. If a quadratic equation in one variable has two real solutions, then
    a) The discriminant of the equation is negative.
    b) The discriminant of the equation is 0.
    c) The discriminant of the equation is positive.
    d) The graph of the equation is a line in the Cartesian coordinate system.
    e) The graph is a point on the real number line.
    f) The graph is two points on the real number line.

37. (1 pt)The graph of an equation in one 

variable is

What is the solution set for that equation?

$\left\{1, 3, 4, \frac{9}{2}\right\}$

38. (3 pt)Complete the statement of The Law of Trichotomy
If a, b, are real numbers then exactly one of the following is true:

i. a < b
ii. a = b
iii. a > b

39. (1 pt)Complete The Transitive Property of equality.
If a, b, and c are real numbers such that a = b and b = c, then a = c.

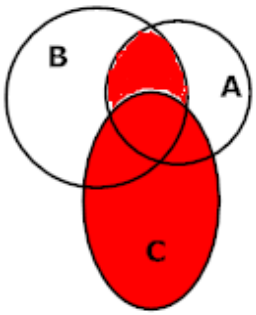
40. (1 pt)Complete the statement of The Zero Factor Property

If a, b, are Real numbers such that ab = 0 then a = 0 or b = 0

41. (2 pt)Calculate the sum $\frac{5}{2x+8}+\frac{7}{3x+12}$. Show your work. Express your answer in simplest terms.

$$\frac{5}{2x+8}+\frac{7}{3x+12}=\frac{5}{2(x+4)}+\frac{7}{3(x+4)}=\frac{15}{6(x+4)}+\frac{14}{6(x+4)}=\frac{29}{6(x+4)}$$

42. (1 pt)In the Venn diagram at the right, shade $(A \cap B) \cup C$



43. (2 pt)Compute the multiplicative inverse of $7 - 8i$

$$\frac{7+8i}{7^2+8^2} = \frac{7+8i}{113}$$

44. (3 pt)Compute the product $(2 - i)(-3 + 5i)$ $= -6 + 10i + 3i - 5i^2 = -1 + 13i$

45. (1 pt)A particular quadratic equation in one variable is solved with the quadratic formula.

One solution is $\frac{3+\sqrt{17}}{12}$. What is the other solution?

$$\frac{3-\sqrt{17}}{12}$$

46. (3 pt)Solve the equation $3x + 4 = -x + \sqrt{2}$

$3x + 4 = -x + \sqrt{2}$

$3x + x = \sqrt{2} - 4$

$4x = \sqrt{2} - 4$

$x = \frac{\sqrt{2}-4}{4}$

The solution set is { $\frac{\sqrt{2}-4}{4}$ }

This is a linear equation in one variable. It is the easiest kind of equation to solve. The fact that one of the coefficients is an irrational number coes not change the equation type.

47. (3 pt)Complete the following process of solving a quadratic equation in one variable by filling in the blanks. **Words are important!**

$x^2 - 8x - 20 = 0$
$(x + 2)(x - 10) = 0$

By The Zero Factor Property

$x + 2 = 0$ OR $x - 10 = 0$
$x = -2$ or $x = 10$
The solution set for $x^2 - 8x - 20 = 0$ is $\{-2, 10\}$

48. (3 pt)A cone has radius 3 feet and volume 36π cubic feet. What is the height? Answer this question by filling in the blanks.

The relevant formula here is the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.

Substitute the given information to obtain $36\pi = \frac{1}{3}\pi 3^2 h$ which we must solve for h.

$36\pi = \frac{1}{3}\pi(3)^2 h$ $36\pi = 3\pi h$ $h = \frac{36\pi}{3\pi}$ $h = 12$	Some Comments: You should be alert enough to observe that $\frac{1}{3}3^2 = 3$. Pay attention ot what you are doing. Don't automatically "multiply out" at every opportunity. Generally multiplication will complicate the computations whereas factoring will almost always simplify the work.

Therefore we conclude that the cone is 12 feet high.

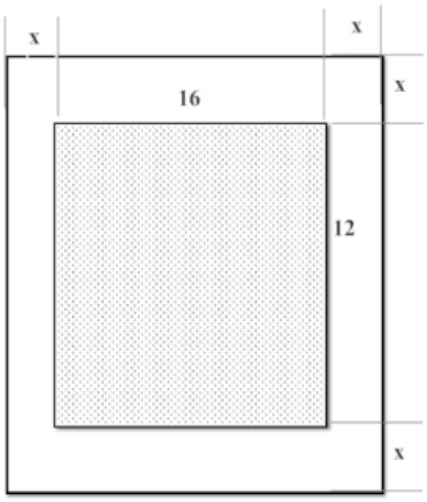
49. (3 pt)Use the quadratic formula to solve the equation $2x^2 - 3x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)} = \frac{3 \pm \sqrt{9 - 32}}{4}$$
$$= \frac{3 \pm \sqrt{-23}}{4} = \frac{3 \pm i\sqrt{23}}{4}$$

A Comment: Note the use of the = symbol! Not the line never ends with a symbol. Because of the Transitive Property there is no need to rewrite the final numbers in some other place on the page.

50. (3 pt)A garden measuring 12 feet by 16 feet is to have a pedestrian pathway installed around it, increasing the total area to 396 square feet. What will be the width of the pathway? You are expected to make a sketch and label it. You are also expected use words to define the variables, give a reason for the equation/model you obtain, and write your conclusion. No credit for guessing! BTW your work shoud show that the path must be 3 feet wide. Just in case you are rusty on multiplication facts (3)(17) = 51.

Let x be the width of the sidewalk. Refer to sketch at the right.



The length of the complete structure is 2x + 16.

The width of the complete structure is 2x + 12.

The area of the complete structure is (2x + 12)(2x + 16)

The area of the complete structure is 396.

We have two expressions for the same quantity.

Therefore by the Transitive Property of equality they must be equal.

This yields the equation (2x + 12)(2x + 16) = 396 which we will solve to answer the question.

(2x + 12)(2x + 16) = 396

4(x + 6)(x + 8) = 396

(x + 6)(x + 8) = 99

$x^2 + 14x + 48 = 99$

$x^2 + 14x - 51 = 0$

(x + 17)(x – 3) = 0

By the Zero Factor Property

x +17 = 0 OR x – 3 = 0

x = –17 OR x = 3

–17 cannot be a length.

Therefore the sidewalk must be 3 feet wide.

A Comment: These geometry based problems are so standard that your presentation should vary from the above in only VERY MINOR ways.

51. (3 pt)Solve the equation $\sqrt{2x-1} = x-2$

$\sqrt{2x-1} = x-2$

$2x-1 = x^2-4x+4$

$x^2-6x+5 = 0$

$(x-5)(x-1) = 0$

By The Zero Factor Property

$x-5 = 0$ OR $x-1 = 0$

$x = 5$ OR $x = 1$

Test 5:

$\sqrt{2(5)-1} = 5-3$ is TRUE

Test 1:

$\sqrt{2(1)-1} = 1-2$ is FALSE

The solution set for the original equation is $\{5\}$.

52. (3 pt)Solve the equation $\frac{x}{x-3} = \frac{3}{x-3} + 9$

$\frac{x}{x-3} = \frac{3}{x-3} + 9$ multiply both sides of the equation by $x - 3$

$x = 3 + 9(x-3)$

$x = 3 + 9x - 27$

$-8x = -24$

$x = 3$

Because 3 causes a zero in a denominator, it is not a solution.

Because 3 is the only possible solution we conclude that the solution set for the original equation is the empty set $\varnothing$

53. (2 pt)Suppose α and β are multiplicative inverses.

a. Then $\alpha\beta =$ **1**

b. The solution set for multiply both sides of the equation by $x - 3$ is 7β

A Comment: See Question 21. Almost everyone got Question 21 correct and almost no one got this one correct. THEY are the same.

If you recall, Mr. Brunk told us 4 or 5 times on each of Wednesday and Friday of last week that to solve an equation of the form $\alpha x = 7$ we must multiply both sides of the equation by the multiplicative inverse of α. So in this case we multiply both sides by β to obtain the simplest equation $x = 7\beta$. Therefore the solution set is $\{7\beta\}$.