College Algebra **TEST 2 Solution** **Spring 2013**

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average_____

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 30 are 1 point each. Others are each 4 points unless otherwise labeled.

1. T F $|3x - 5| = 0$ if and only if $3x - 5 = 0$.
2. T F A vertical line is not the graph of a function.
3. T F The interval (3, 7] contains irrational numbers.
4. T F If a vertical line intersects a graph, that graph is not the graph of a function.
5. T F The solution set of $|x + 7| < 2$ can be written using the roster method.
6. T F A linear function is a function whose rule can be written in the form $f(x) = mx + b$.
7. T F If a horizontal line intersects a graph that graph may be the graph of a function..
8. T F The graph of $x^2 + y^2 < 5$ is the inside of a circle.
9. What is the solution set for $|x^2 - 5x + 6| < -3$? $\varnothing$
10. When considering an equation and its two corresponding inequalities, we refer to the equation as the boundary equation.
11. The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

12. The midpoint of the line segment joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

13. The equation of a circle with radius r and center at the origin is:

$$x^2 + y^2 = r^2$$

14. The equation of a circle with radius r and center at the point (h, k) is:

$$(x - h)^2 + (y - k)^2 = r^2$$

15. The slope of the line through two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

16. Write the equation of the circle with radius 5 and center (–3, 8). $(x + 3)^2 + (y - 8)^2 = 5^2$
17. If a point is on the x-axis, then its second coordinate is 0.
18. The rule for the linear function whose graph has slope 5 and y-intercept 2 is $f(x) = 5x + 2$.

19. The graph of a function is the set of all points of the form (a, f(a))

    where a is an element of the domain and f(a) is the corresponding range element.

20. A quadratic function is a function whose rule may be written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and a is not zero.

21. According to The Transitive Property, if two expressions represent the same quantity, then those two expressions are equal.

22. The sum of two functions f and g with the same domain is the function named (f+g) whose rule may be written as $(f+g)(x) = f(x) + g(x)$ for all x in the common domain.

23. Write the inequality $|2x - 4| < 8$ as a compact compound inequality. $-8 < 2x - 4 < 8$

24. The solution set for $|3x + 7| = -4$ is $\varnothing$.

25. If f and g are functions and the rule for g is $g(x) = f(x) + 4$, then the graph of g is the same as the graph of f shifted (up, down, left, right). Circle the correct choices.

26. If f and g are functions and the rule for g is $g(x) = f(x+4)$, then the graph of g is the same as the graph of f shifted (up, down, left, right). Circle the correct choices.

27. If f and g are functions and the rule for g is $g(x) = f(x+4) + 4$, then the graph of g is the same as the graph of f shifted (up, down, left, right). Circle the correct choices.

28. The graph of a quadratic function is a parabola which opens up if a is positive.

29. The fact that (8, 3) is on the graph of a function f means $f(8) = 3$.

30. Unless otherwise stated, the domain of a function is the largest set of real numbers for which the rule makes sense (has meaning)

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices.
For Questions 31 – 36 each part is worth one point

31. If x and y are real numbers which of the following are true.
    a. $|x| > 0$.
    b. $|xy| = |x||y|$.
    c. If k is a real number, then $|x| < k$ is equivalent to $-k < |x| < k$.
    d. If k is a positive real number, then $|x| < k$ is equivalent to $-k < |x| < k$.
    e. If k is a positive real number, then $|x| > k$ is equivalent to $-k > |x| > k$.
    f. $|x - y| = |y - x|$.

32. If the point (5, –7) is on the graph of a function f, which of the following are true:
    a. $f(-7) = 5$
    b. $f(5) = -7$
    c. The rule for f is $f(x) = 5x - 7$
    d. 5 is in the domain of f

33. Complete the following algebraic definition of absolute value.

The absolute value of a real number x or an expression x which represents a real number is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

34. Consider the circle whose equation is $x^2 + y^2 + 2x + 12y - 12 = 0$.

a. Write this equation in Standard Form

$x^2 + y^2 + 2x + 12y - 12 = 0$

$x^2 + 2x + y^2 + 12y = 12$

$(x^2 + 2x + 1) + (y^2 + 12y + 36) = 12 + 1 + 36$

$(x + 1)^2 + (y + 6)^2 = 7^2$

b. What is the center of the circle? $(-1, -6)$

c. What is the radius of the circle? 7

35. The graph of a particular absolute value inequality of the form $|ax + b| < k$ is 

a. Use interval notation to write the solution set for $|ax + b| > k$

$(-\infty, 3) \cup (7, +\infty)$

b. Use the roster method to write the solution set for $|ax + b| = k$ $\{3, 7\}$

36. The solution sets for the three statements $|ax + b| < k$, $|ax + b| = k$, $|ax + b| > k$, are shown in red, green and blue.

a) The solution set for $|ax + b| > k$ is shown in red.

b) The solution set for $|ax + b| = k$ is shown in green.

c) The solution set for $|ax + b| < k$ is shown in blue.

37. Calculate the midpoint of the line segment joining (–3, – 4) and (– 2, 7).

The midpoint is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3-2}{2},\frac{-4+7}{2}\right)=\left(\frac{-5}{2},\frac{3}{2}\right)$

38. Calculate the distance between (3, – 4) and (– 2, – 7).

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(3+2)^2+(-4+7)^2}=\sqrt{5^2+3^2}=\sqrt{25+9}=\sqrt{34}$

39. Find the solution set for the inequality $|3x-8|>4$? Show the steps of your solution process. Write the solution set in interval notation. **Words are an important part of your work. A graph would also help.**

We will begin by solving the easy inequality $|3x-8|<4$ which is equivalent to

$-4<3x-8<4$

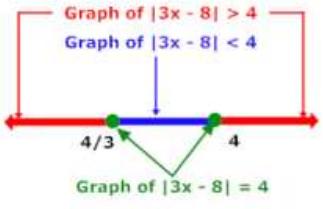
$4<3x<12$

$\frac{4}{3}<x<4$

The solution set for $|3x-8|<4$ is the interval $\left(\frac{4}{3},4\right)$

It then follows that the solution set for $|3x-8|>4$ is everything outside this interval excluding the endpoints.

So the solution set for $|3x-8|>4$ is $\left(-\infty,\frac{4}{3}\right)\cup(4,+\infty)$



40. What is the domain of the function f whose rule is $f(x)=\frac{\sqrt{x+5}}{x-2}$ Write your response in interval notations and include a graph.

This rule only makes sense if $x+5\geq 0$ or equivalently $x\geq -5$ and if $x\neq 2$

Therefore the domain of f is $[-5,2)\cup(2,+\infty)$

41. Find the rule for the linear f function whose graph contains the two points (3, 5) and (7, –1).

Because the desired function is linear, its rule must be of the form f(x) = mx + b

$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5+1}{3-7} = -\frac{3}{2}$

Therefore f has the form $f(x) = -\frac{3}{2}x + b$

Because $(3,5)$ is on the graph of f, $f(3) = 5$

However $f(3) = -\frac{3}{2}(3) + b = -\frac{9}{2} + b$

From the Transitive Property it follows that $-\frac{9}{2} + b = 5$

Therefore $b = 5 + \frac{9}{2} = \frac{10+9}{2} = \frac{19}{2}$

The rule for the desired function is $f(x) = -\frac{3}{2}x + \frac{19}{2}$

42. Show the point $\left(2, \frac{4}{7}\right)$ is on the graph of the function whose rule is $f(x) = \frac{3x-2}{x+5}$ Use correct function notation.

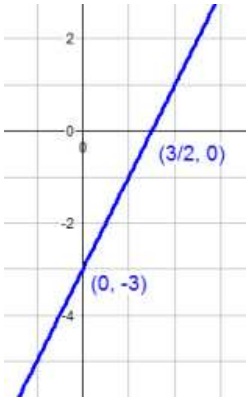
$f(2) = \frac{3(2)-2}{2+5} = \frac{4}{7}$

It now follows from the definition of graph of a function that the point $\left(2, \frac{4}{7}\right)$ is on the graph of f.

43. Sketch the graph of the function whose rule is
f(x) = 2x – 3. Show all computations. Label important points with their coordinates.

f is a linear function so its graph is a non-vertical line.
f(0) = –3 implies the y-intercept is (0, –3)

f(x) = 0 is equivalent to 2x – 3 = 0 which is equivalent to $x = \frac{3}{2}$



44. **(5 points)** Complete the following analysis by filling in the blanks:
Question: How much water must be added to 20 ounces of a 15% acid solution to reduce it to a 10% acid solution?
Analysis: Let x be the amount of water to be added.
The amount of the final solution is x + 20.

The amount of acid in the original solution is **(0.15)(20)** = 3.

The amount of acid in the final solution is **3**.

The amount of acid in the final solution is **(0.10)(x + 20)** = 0.1x + 2.

We now have two expressions for the same quantity.

According to The Transitive Property these two expressions must be equal.

Therefore **0.1x + 2 = 3**.

This is the model for the question and the answer to the question is obtained by solving the equation.

The solution for the equation is {10}. **Trust me that's what it is: you don't need to solve the equation.**

Therefore **10 ounces** of water must be added to obtain a 10% acid solution.

45. **(6 points)** Definition: A **function** consists of three things;

- A set called the **domain**
- A set called the **range**
- A **rule** which associates **each** element of the **domain** with a **unique** element of the range.

(4 Points)