College Algebra **TEST 3 Solution** **Spring 2013**

NAME: ______________________________ Score_____________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average__________

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 40 are 1 point each. Others are each 5 points unless otherwise labeled.

1. T F Every polynomial is a term.
2. T F Every function has an inverse.
3. T F The graph of every rational function has a vertical asymptote.
4. T F The graph of a rational function with a vertical asymptote can cross (intersect) that asymptote.
5. T F The composition of two functions is a function.
6. T F A term is a letter, a number, or a product of letters and numbers.
7. T F If f and g are functions and f is the inverse of g, then g is the inverse of f.
8. T F Every trinomial polynomial is a quadratic polynomial.
9. T F The graph of a rational function cannot intesect its horizontal asymptote.
10. T F The graph of a quadratic function is a parabola.
11. T F A polynomial is a letter, a number, or a product of letters and numbers.
12. T F If 3 + 2i is a complex zero of a polynomial function, then so is 3 – 2i.
13. T F It is possible for the entire graph of a rational function to be above the x-axis.
14. T F It is possible for the entire graph of a polynomial function to be above the x-axis.
15. T F A quadratic function is a polynomial function.
16. Complex zeros of polynomial functions occur in conjugate pairs.
17. The numerical part of a term is called the coefficient of the term.
18. If a real zero of a polynomial has odd multiplicity, then the graph crosses the x-axis at that zero.
19. If the leading coefficient of a fifth degree polynomial function is –7 then as $x \to -\infty, f(x) \to +\infty$.
20. If the rule for a polynomial function is $f(x) = (x-2)^3(x-4)$ then 2 has multiplicity 3.
21. The graph of a polynomial function is a smooth continuous curve with no sharp corners.
22. If x – 5 is a factor of a polynomial function, then (5,0) is an x-intercept of the graph of f
23. A polynomial is a term or a sum of terms in which all variables have whole number exponents.
24. A polynomial consisting of two terms is called a binomial.
25. If $\frac{3}{5}$ is a zero of a polynomial with integer coefficients, then 3 is a divisor of the constant term.
26. The composition of a function f with a function g is a function named $f \circ g$ whose rule is $f \circ g(x) = f(g(x))$

27. The symbol f^{-1} is read as **f inverse.**

28. The coefficient of the term $-4x^5$ is -4

29. The degree of $-4x^8$ is 8

30. A quadratic function is a function whose rule can be written in the form $f(x) = ax^2 + bx + c$

31. The x–intercepts of the graph of f(x) = (x – 3)(x + 1)(2x + 5) are 3, –1, –5/2

32. The point (3, f(3)) is on the graph of f and if f(3) > 0, the point is above the x-axis.

33. The rule for the vertex of the graph of the quadratic function f is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

34. The graph of a quadratic function is a parabola.

35. What is the discriminant of the quadratic function whose rule is $f(x) = x^2 - 4x + 5$?

$(-4)^2 - (4)(1)(5) = 16 - 20 = -4$

36. What is the x-intercept of $f(x) = \frac{3x-5}{x+2}$ $\left(\frac{5}{3}, 0\right)$

Because x-intercepts occur at real zeros of the numerator.

37. What is the vertical asymptote of $f(x) = \frac{3x-5}{x+2}$ $x = -2$

Because vertical asymptotes are lines and occur at real zeros of the denominator.

38. What is the horizontal asymptote of $f(x) = \frac{3x-5}{x+2}$ $y = 3$

Because numerator and denominator have the same degree. Asymptotes are lines.

39. What is the horizontal asymptote of $f(x) = \frac{5x-4}{x^2}$ $y = 0$

Because degree of numerator is less than the degree of the denominator. Asymptotes are lines.

40. Solve $\frac{x-5}{x^2+1} > 0$.

$\frac{x-5}{x^2+1} > 0$ iff $x - 5 > 0$ iff $x > 5$. So the solution set is $(5, +\infty)$

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices.

For Questions 41 – 44 each part is worth one-half point

41. Which of the following are true about the function whose rule is $f(x) = \frac{(x-1)(x-3)}{(x-5)}$
    - a. f is a rational function
    - b. f (x) is a rational function
    - c. f(x) is a range element
    - d. x is a domain element
    - e. the line x =1 is a vertical asymptote
    - f. the line x =3 is a vertical asymptote
    - g. the line x =5 is a vertical asymptote
    - h. the x-axis is a horizontal asymptote
    - i. the line y =1 is a horizontal asymptote
    - j. there is no horizontal asymptote

42. Which of the following are the rules for polynomial functions
    - a. $f(x) = 3x + 5$
    - b. $f(x) = 3x^2$
    - c. $f(x) = 3x^2 - 5x + 12x^{-1}$
    - d. $f(x) = \frac{3}{4}x^5 - 2x^3 + \sqrt{5}x - \frac{8}{\sqrt{3}}$
    - e. $f(x) = \frac{x^2 + 2x - 5}{x}$
    - f. $f(x) = x^7 - 5$
    - g. $f(x) = (x-2)^4(x+6)$
    - h. $f(x) = \frac{x+1}{x-1}$
    - i. $f(x) = \frac{x^2 + 3x - 4}{5}$
    - j. $f(x) = \frac{5}{x^2 + 3x - 4}$

43. Circle the letters to indicate which of the following are terms.
    - a) 4
    - b) $x^4 - 4x^3 + 1$
    - c) $\frac{3}{4}x^7$
    - d) x^2
    - e) $3x + 5$

44. If a quadratic function in two variables has two x-intercepts, then
    - a. The discriminant of the function is negative.
    - b. The discriminant of the function is 0
    - c. The discriminant of the function is positive
    - d. The graph of the function is a line.
    - e. The graph of the function opens down.

45. Suppose $f(x) = 7x + 1$ and $g(x) = 2x^2 - 9$. Calculate the rule for the function $f \circ g$

$$f \circ g(x) = f(g(x)) = f\left(2x^2 - 9\right) = 7\left(2x^2 - 9\right) + 1 = 14x^2 - 62$$

46. If f is a polynomial function whose rule is $f(x) = -3x^5 + 3x^4 - 5x - 34$ then

As $x \to +\infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to +\infty$

47. The function whose rule is $f(x) = 5x - 2$ is a linear function and therefore it has an inverse f^{-1}. Find the rule for f^{-1}. Use the normal 5 step process.

$f(x) = 5x - 2$

$y = 5x - 2$

$x = 5y - 2$

$y = \frac{x+2}{5}$

$f^{-1}(x) = \frac{x+2}{5}$

48. What are the possible rational zeros of $f(x) = 3x^5 + 8x^3 + 4x - 2$.

$$p \in \{\pm 1, \pm 2\}$$

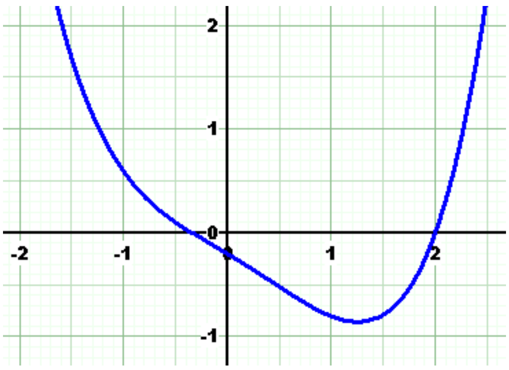
$$q \in \{\pm 1, \pm 3\}$$

$$\frac{p}{q} \in \left\{\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}\right\}$$

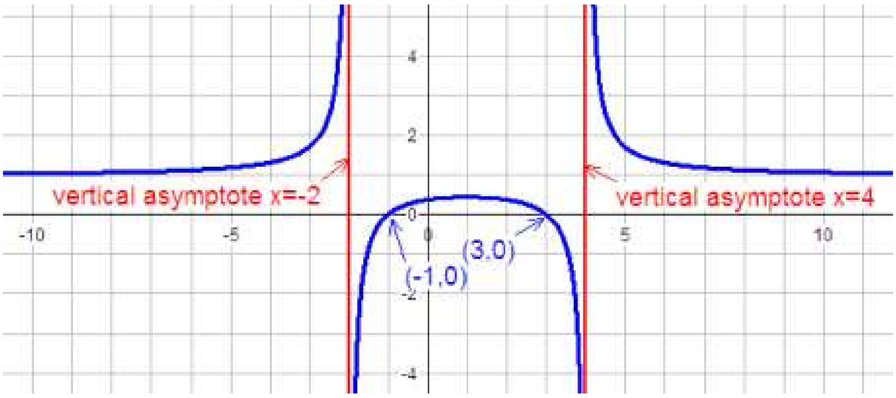
49. The possible rational zeros of the function whose rule is $f(x) = 3x^4 - 2x^3 - x^2 - 12x - 4$ are calculated to be $\frac{p}{q} \in \left\{\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 4, \pm\frac{4}{3}\right\}$

A computer generated graph of f is shown at the right.
Use that graph to determine which of the possible rational zeros are reasonable guesses that should be tested. DO NOT TEST THEM.

The set of reasonable choices is $\left\{2, -\frac{1}{3}\right\}$



50. The graph of the function f whose rule is $f(x) = \frac{(x+1)(x-3)}{(x+2)(x-4)}$ is shown below. Use that graph (nothing else—no computations) to answer the questions listed below the graph. Use interval notation or roster method to state your answer.



a. The solution set of $\frac{(x+1)(x-3)}{(x+2)(x-4)} < 0$ is $(-2,-1) \cup (3,4)$

b. Where is f(x) > 0? $(-\infty,-2) \cup (-1,3) \cup (4,+\infty)$

c. What is the set of real zeros of f ? $\{-1,3\}$

51. The rule for particular quadratic function has the form $f(x) = x^2 - 4x + c$ where c is a real number that is determined by the application. In a particular application, it is found that f(2) = 5. In this case what is the value of c?

Using the rule for f to calculate f(2) we obtain $f(2) = 2^2 - 4(2) + c = -4 + c$

We now have two expressions for f(2). They must be equal.

$-4 + c = 5$ which implies $c = 9$.

52. Sketch the graph of the function whose rule is $f(x)=\dfrac{x+5}{x-7}$ Show your work neatly.

f is a rational function.

x-intercepts occur at the real zeros of the numerator. f has an x-intercept at (-5,0).

Vertical asymptotes occur at the real zeros of the denominator. The vertical asymptote for f is x = 7.

Because the numerator and denominator have the same degree, the line y = 1 is a horizontal asymptote.

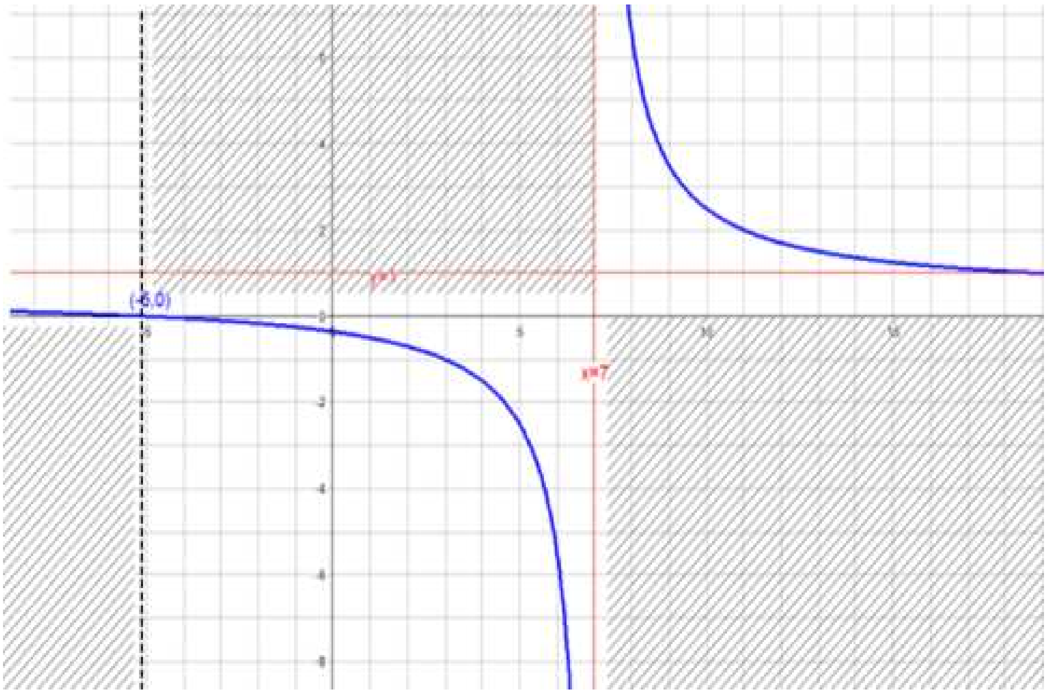
The equation $\dfrac{x+5}{x-7}=1$ is equivalent to x +5 = x – 7 which is clearly a contradiction. Therefore the graph of f does not cross its horizontal asymptote.

Test -6, 0, and 8.

$$f(-6)=\frac{-6+5}{-6-7}>0$$

$$f(0)=\frac{0+5}{0-7}<0$$

$$f(8)=\frac{8+5}{8-7}>0$$



53. Sketch the graph of the function whose rule is $f(x) = -(2x-3)(x+2)(x-5)$ Show your work neatly.

f is a third degree polynomial function .
Therefore its graph is a smooth continuous curve with no sharp corners and it tries to cross the x-axis three times.

Clearly f has three x-intercepts (3/2, 0), (-2, 0), and (5, 0).

The leading term $-2x^3$ dominates when x is far from the origin. Therefore
$$\text{As } x \to +\infty, f(x) \to -\infty$$
$$\text{As } x \to -\infty, f(x) \to +\infty$$