College Algebra **TEST 1 Solution** **Spring 2014**

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average_____

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 40 are $\frac{1}{2}$ pts each

1. **T** F A linear equation in one variable is an equation which can be written in the form mx + b = 0.
2. T **F** Addition is a unary operation.
3. T **F** The solution set for a linear identity is the empty set.
4. **T** F Two equations are equivalent if they have the same solution sets.
5. T **F** A real number which is not an integer is irrational.
6. **T** F The solution set of |4x + 5| < 2 is an interval.
7. **T** F The interval [–2, 0] contains rational numbers and irrational numbers.
8. T **F** The solution set for the equation 3x = 12 is 4.
9. T **F** The set of Rational Numbers is a subset of the set of Integers.
10. **T** F |3x – 5| = 0 if and only if 3x – 5 = 0.
11. T **F** The inequality |3x – 7| > 2 can be written as the compact compound inequality – 2 > 3x – 7 > 2.
12. **T** F If 3x + 5 is added to both sides of an equation, the new equation is equivalent to the original equation.
13. T **F** The solution set for |3x + 9| = – 5 is **R**.
14. T **F** To say 3 is a solution of an equation means the same as saying {3} is the solution set for that equation.
15. **T** F An equation is a mathematical statement which contains an = symbol.
16. T **F** The norm of 3 – 4i is $3^2 - 4^2$.
17. T **F** When both sides of an equation are multiplied by an expression containing a variable the resulting equation will be equivalent to the original.
18. **T** F The product of a complex number and its conjugate is its norm.

19. A formula must be an **equation**.

20. A binary relation is a **comparison** of two operands.

21. Definitions in mathematics are **stipulative**.

22. The graph of an equation in **one** variable is drawn on the the Real number line.

23. The graph of an equation consists of all the points, and only those points, whose coordinates are **solutions** of the equation.

24. A conditional equation is an equation which is **true** when some real numbers are substituted for the variables and is **false** when some real numbers are substituted for the variables.

25. Write the conjugate of 9 – 4i. **9 + 4i**

26. What is the complex component of $\frac{4}{3} + \frac{5}{9}i$ $\frac{5}{9}$

27. The solution set for a linear identity in one variable is **R**

28. The solution set for $|5x + 6| < -4$ is $\varnothing$

29. The process to solve a linear equation in one variable is to generate a sequence of equations each **equaivalent** to the previous equation until a simplest equation is obtained.

30. Write the formula for the area of a triangle. $A = \frac{1}{2}bh$

31. Write the inequality $|5x - 2| < 1$ as a compact compound inequality. **$-1 < 5x - 2 < 1$**

32. The solution set for $|3x + 7| = -4$ is $\varnothing$

33. Write the formula for the area of a circle. $A = \pi r^2$

34. A certain trapezoid has height h = 5 with bases B = 3 and b = 7. What is its area? **25**

35. Use interval notation to write the set $\{x \mid -2 < x < 5\}$ **= (–2, 5)**

36. A **term** is a letter, a number, or a product of letters and numbers.

37. The discriminant of a quadratic $ax^2 + bx + c$ is **$b^2 - 4ac$**

38. The phrase "to factor" means to write as a **product**.

39. Transitive Property: If a, b, and c are real numbers such that a = b and b = c, then **a = c**.

40. Consider the equation $\sqrt{3x + 6} = x + 9$ with solution set A.
Also consider the equation $3x + 6 = (x + 9)^2$ with solution set B.
What is the relation between the solution sets A and B? $A \subset B$ **(or B contains A)**
Hint: No computations are required nor expected.

For Questions 41 – 51 each part is worth one point

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices.

41. Which of the following are linear equations in one variable?

a. $3x + 7 < 2x + 12$

b. $3x + 7 < 2y + 12$

c. $\sqrt{7x} + 1 = 0$

d. $\sqrt{7}x + 1 = 0$

e. $x^2 = 3$

f. $x + 5$

g. $\frac{2x}{5} + \sqrt{3} = \frac{3}{\sqrt{2}}x$

h. $\frac{x - 3}{4} = \frac{2 - 3x}{9}$

i. $\frac{x - 3}{4} = \frac{2 - 3x}{x}$

j. $\sqrt{8} = x$

k. $\sqrt{3x + 1} + 8 = x$

42. Which of the following are rational equations in one variable?

a. $3x - 7 = 2x + 12$

b. $\sqrt{7x} + 1 = 0$

c. $\frac{x}{3x+5} = 7$

d. $\frac{2x}{5} + \sqrt{3} = \frac{3}{\sqrt{2}}x$

e. $\frac{x-3}{4} = \frac{2-3x}{x}$

f. $\frac{x}{5} = \frac{7x-11}{2}$

g. $\sqrt{8} = x$

43. The graph of a conditional linear equation in one variable:

a. Is on the number line

b. Is in the Cartesian coordinate system

c. Is a line

d. Is a dot

e. Is a collection of more than one dot

f. Is an interval

g. Is a ray

44. If both sides of an inequality are multiplied by a (the same) positive real number:

a. The resulting inequality is equivalent to the original inequality.

b. The two inequalities have the same solution sets.

c. The two inequalities might have different solution sets.

d. The two inequalities are equal.

e. The resulting inequality is a simplest inequality.

45. Insert the correct symbol in the $\square$ to make the statement true.

You should choose from these symbols: $\in \quad \notin \quad \subset \quad \not\subset \quad = \quad \neq \quad < \quad >$

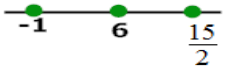
a. $9 \ \boxed{\in} \ \{x|x \text{ is an integer}\}$

b. $\{3\} \ \boxed{\subset} \ \{x|x \text{ is an integer}\}$

c. $-4 \ \boxed{\notin} \ \{x|x \text{ is an irrational number}\}$

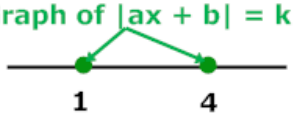
d. $\sqrt{3} \ \boxed{\notin} \ \{x|x \text{ is a rational number}\}$

46. The graph of an equation in one variable is



What is the solution set for that equation? $\left\{-1, 6, \frac{15}{2}\right\}$

47. The graph of a particular absolute value equation of the form $|ax + b| = k$ is



a. Use interval notation to write the solution set for $|ax + b| < k$ **(1, 4)**

b. Use the roster method to write the solution set for $|ax + b| = k$ **{1, 4}**

c. Use interval notation to write the solution set for $|ax + b| > k$ **$(-\infty, 1) \cup (4, \infty)$**

48. The solution sets for the three statements $|ax + b| < k$, $|ax + b| = k$, $|ax + b| > k$, are shown in red, green and blue.

a) The solution set for **$|ax + b| > k$** is shown in red.

b) The solution set for **$|ax + b| = k$** is shown in green.

c) The solution set for **$|ax + b| < k$** is shown in blue

49. Complete the following **algebraic** definition of absolute value.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

50. Fill in the blanks in the following process to solve $x^2 - 5x - 6 = 0$

$x^2 - 5x - 6 = 0$

$(x + 1)(x - 6) = 0$

Then by the **ZERO FACTOR PROPERTY**

$x + 1 = 0$ OR $x - 6 = 0$

$x = -1$ OR $x = 6$

The solution set is **{-1, 6}**

51. Complete the statement of The Law of Trichotomy

If a and b are real numbers then exactly one of the following is true:

i. **$a < b$**
ii. **$a = b$**
iii. **$a > b$**

Problems 52 – 57 are each worth 5 points.

52. Compute the multiplicative inverse of $5 - 3i$.

The multiplicative inverse is $\dfrac{5 + 3i}{5^2 + 3^2} = \dfrac{5 + 3i}{34}$

The multiplicative inverse of a complex number is its conjugate divided by its norm.

53. Compute the product $(2 + i)(3 - 5i)$

$(2 + i)(3 - 5i) = (2)(3) - (2)(5i) + 3i - 5i^2 = (6 + 5) - (-10 + 3)i = 11 - 7i$

NOTE: Observe the use of the = symbol. Look at your author's examples on pp. 131 -135 and observe the use of equal symbols. You MUST use equal symbols to indicate two expressions are equal. The reader is NEVER expected to decide the relation between two expressions.

You can use either the horizontal method (as above) or the vertical method (as in text) for displaying your work but it must NEVER be a random collection of scribbles.

Some students attempted to end this computation with a written conclusion. That is a good idea. However, to say that 11 – 7i is the solution or that {11 – 7i} is the solution set is not correct. Solutions and solution sets are results of solving equations or inequalities. Nothing else.

A correct conclusion would be the following:

The product of (2 + i) and (3 – 5i) is 11 – 7i.

54. Solve the inequality $2x - 5 < 5x + 4$. Write the solution set in interval notation.

$2x - 5 < 5x + 4$
$-3x < 9$
$x > -3$
The solution set is the interval $(-3, \infty)$.

55. Solve the equation $\frac{x-2}{x+3} = 4$

$\frac{x-2}{x+3} = 4$
$x - 2 = 4(x + 3)$ This equation might not be equivalent to the original
$x - 2 = 4x + 12$
$-14 = 3x$
$x = -\frac{14}{3}$
This is the only possible solution and because it does not create a zero in any denominator of the original equation it is a solution.
Therefore $\left\{-\frac{14}{3}\right\}$ is the solution set for the original equation.

The solution set is $\{ -\frac{14}{3} \}$

56. Use the quadratic formula to solve the equation $x^2 + 2x + 3 = 0$ Simplify as much as possible
Steps that can reasonably be omitted are shown in blue. The other steps AND THE EQUAL SYMBOLS should be included in a response to this question.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 12}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm \sqrt{(4)(-2)}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{-2}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-2}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = \frac{\cancel{2}(-1 \pm i\sqrt{2})}{\cancel{2}} = -1 \pm i\sqrt{2}$$

57. Find the solution set for the inequality $|2x + 8| > 4$? Show the steps of your solution process. Write the solution set in interval notation. **Use the process discussed in class. That means – Solve the easy one first then DEDUCE the desired solution. No credit for any other process. Words are an important part of your work. A graph would also help.**

solve the easy inequality $|2x + 8| < 4$ and deduce from its solution set the solution set for $|2x + 8| > 4$

$|2x + 8| < 4$ is equivalent to the compound compact inequality
$-4 < 2x + 8 < 4$
$-12 < 2x < -4$
$-6 < x < -2$
The solution set for $|2x + 8| < 4$ is the interval $(-6, -2)$.
Therefore the solution set for $|2x + 8| > 4$ is $(-\infty, -6) \cup (-2, \infty)$