College Algebra **TEST 2 Solution** **Spring 2014**

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average_____

No Decimals, mixed numbers, complex fractions, and boxed or circled answers. Use function notation. Show work. If you use a formula – state the formula, then use it.

Questions 1 – 20 are 1 pt each

1. T **F** The slope of the line through two points (x_1, y_1) and (x_2, y_2) is given by the formula $m = \frac{x_1 - x_2}{y_1 - y_2}$

2. **T** F To find the x-intercepts of a graph of a function f we let f(x) = 0 and solve for x.

3. **T** F The reciprocal function is the function f whose rule may be written in the form $f(x) = \frac{1}{x}$.

4. **T** F A zero of a function f is a domain element k for which f(k) = 0.

5. **T** F A vertical line has no slope.

6. T **F** If the discriminant of a quadratic function is negative and its leading coefficient a is negative then its graph is entirely above the x-axis.

7. **T** F If $b^2 - 4ac = 0$, the equation $y = ax^2 - bx + c$ has exactly one solution.

8. T **F** The second coordinate of every point in Quadrant II of the Cartesian Coordinate System is negative.

9. **T** F The domain of a function is a set.

10. **T** F If a vertical line may be drawn so that it intersects a graph in more than one point, then that graph is not the graph of a function .

11. Unless otherwise stated, the domain of a function is the **largest** set of real numbers for which the rule makes sense (has meaning).

12. The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

13. A quadratic equation in two variables is an equation which can be written in the form **$y=ax^2 + bx + c$**

14. A quadratic function is a function whose rule may be written in the form **$f(x)=ax^2 + bx + c$**

15. What mathematical law dictates that exactly one of the following must b e true about the discriminant of a quadratic function. $b^2 - 4ac < 0$, $b^2 - 4ac = 0$, or $b^2 - 4ac > 0$ **The Law of Trichotomy**

16. The discriminant of a quadratic function $f(x) = ax^2 + bx + c$ is **$b^2 - 4ac$**

17. A point is on the x-axis if and only if its **second** coordinate y is **zero** .

18. When an ordered pair of numbers (a,b) is used to specify a point in the Coordinate plane, the numbers a and b are called **coordinates**

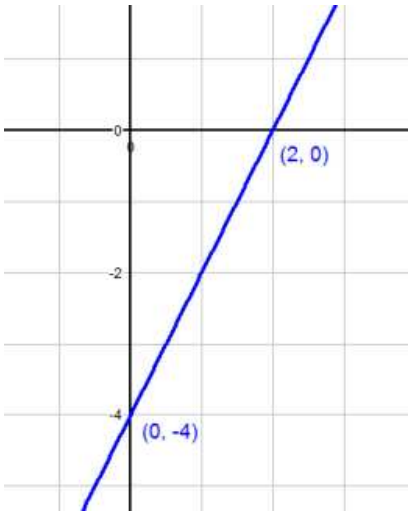
19. The graph of a function f is the set of points of the form **(a, f(a))** .

20. Complete the quadratic formula:

The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

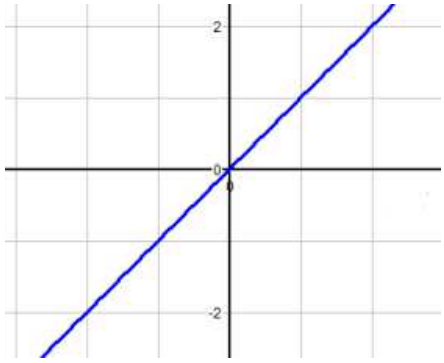
Problems 21 – 30 are 3 points each.

21. Sketch the graph of f(x) = 2x – 4 .



f is a linear function, so its graph is a line.
f(0) = –4 so the y-intercept is (0, –4)
f(x) = 0 imples x = 2 so the x-intercept is (2,0)
Two points determine a line, so you only need to plot two points.
If you plot a whole bunch of points, you are telling me that you do not understand a very basic property of lines.
The two points you should normally use for a line are the intercepts.
♣The y-intercept because it is so easy to determine.
♣The x-axis because almost all answers to questions involve the x-intercept of the graph of an equation or function.

22. Sketch the graph of the identity function .



23. Find the length of the line segment joining (3, –5) and (–2, 4).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3+2)^2 + (-5-4)^2} = \sqrt{(5)^2 + (-9)^2} = \sqrt{106}$$

The following is not acceptable: You cannot drop and reintroduce the radical in some willy-nilly fashion.
If you use a formula, state the formula and then use it.

$$d = \sqrt{(3+2)^2 + (-5-4)^2} = (5)^2 + (-9)^2 = 25 + 81 = \sqrt{106}$$

24. Find the midpoint of the line segment joining (3, –5) and (–2, 4) .

The midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3-2}{2}, \frac{-5+4}{2}\right) = \left(\frac{1}{2}, \frac{-1}{2}\right)$

25. What is the vertex of the graph of the function f(x) = – x^2 – 2x + 3 State the formula – then use it

The Vertex is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) == \left(\frac{2}{-2}, f\left(\frac{2}{-2}\right)\right) = (-1, f(-1)) = (-1, 4)$$

Note : $f(-1) = -(-1)^2 - 2(-1) + 3 = -1 + 2 + 3 = 4$

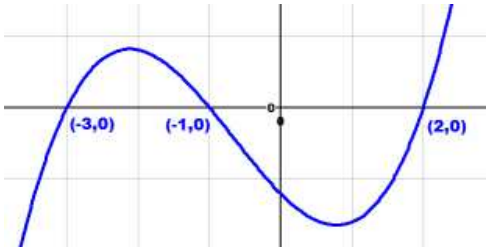
26. What are the x-intercepts of the quadratic function whose rule is $f(x) = x^2 - 9$. Show some work.

Because x-intercepts are on the x-axis they are the x-values for which f(x) =0. So we solve:
$0 = f(x) = x^2 - 9 = (x - 3)(x + 3)$
By The Zero Factor Property x = 3 or x = – 3
The x-intercepts are 3 and – 3

27. What is the domain of the function whose rule is $f(x) = \frac{3x-1}{x+5}$. **Your response should show some computation and should include some words.**

The domain of f is the largest set of real numbers for which the denominator is not zero.
We must exclude all solution to x + 5 = 0
So the domain is all real numbers except – 5
The domain is $\{x \mid x \neq -5\} = (-\infty, -5) \cup (-5, \infty)$

28. The graph of a function f is shown at the right. Use that graph to answer the following three questions. Write your answers using interval notation or the roster method.



a. Where is f(x) < 0 ?
For $x \in (-\infty, -3) \cup (-1, 2)$
b. Where is f(x) = 0 ?
For x ∈ {-3, -1, 2}
c. Where is f(x) > 0 ?
For $x \in (-3, -1) \cup (2, +\infty)$

Because the graph of a function consists of points of the form (x,f(x)) and because a point is:
i. below the x-axis if it second coordinate is negative (f(x) < 0)
ii. on the x-axis if its second coordinate is 0 (f(x) = 0)
iii. above the x-axis if its second coordinate is positive (f(x) > 0)

It follows that:
1. f(x) < 0 if the graph is below the x-axis
2. f(x) = 0 if the graph is on the x-axis
3. (x) > 0 if the graph is above the x-axis

29. What is the x-intercept of the graph of the linear function f whose rule is $f(x) = \sqrt{3}x - \pi$. Note: x is not part of the radicand .

Because x-intercepts are on the x-axis they are the x-values for which f(x) = 0. So we solve the equation resulting from f(x) =0.

$0 = \sqrt{3}x - \pi$

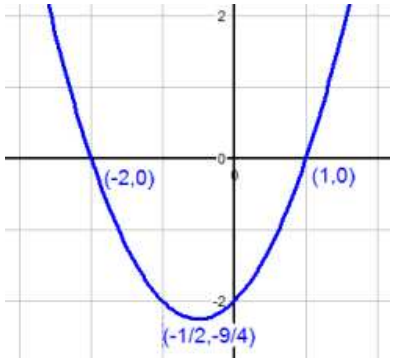
$\pi = \sqrt{3}x$

$x = \frac{\pi}{\sqrt{3}}$

♣ You must accept the fact that there are irrational numbers. You must become proficient at working with irrational numbers. You must resist the temptation to replace an irrational number with some decimal approximation of that number.

Problems 31 – 39 are 5 points each unless otherwise stated.

30. Sketch the graph of the function whose rule is $f(x) = x^2 + x - 2 = (x + 2)(x - 1)$. **Show your work. Label all important points.**



f is a quadratic function so its graph is a parabola.

Because the leading coefficient is positive the parabola opens up.

The fact that x-intercepts occur when f(x) = 0, the factorization, and the Zero Factor Property make it clear the x-intercepts are at –2 and 1.

The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right) = \left(-\frac{1}{2}, -\frac{9}{4}\right)$

Note $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2} + 2\right)\left(-\frac{1}{2} - 1\right) = \left(\frac{3}{2}\right)\left(-\frac{3}{2}\right) = -\frac{9}{4}$

31. Consider the function whose rule is $f(x) = 3x - 9$.

a. f is a linear function

b. The graph of f is a line.

c. The y intercept is f(0).

d. The x-intercept is f(0).

e. The y-intercept of the graph of f is –9.

f. The x-intercept of the graph of f is 3.

g. The y-intercept is 9

h. The x-intercept is 3

i. The x-intercept is –3

j. The slope of the graph is 3

32. Suppose f is a quadratic function whose rule is $f(x) = ax^2 + bx + c$ and whose discriminant is positive. Which of the following is true?

a. f(0) is the y-intercept
b. The equation $y = ax^2 + bx + c$ has exactly two solutions.
c. The graph of f is a parabola
d. The graph of f opens up
e. The graph of f has exactly one x-intercept
f. The graph of f has exactly two x-intercepts
g. The graph of f has no x-intercepts
h. The vertex is on the x-axis
i. The vertex is below the x-axis
j. The vertex is above the x-axis

33. Find the rule for the linear function whose graph passes through the point (5, 4) with slope $\frac{1}{3}$.

Because the desired function is linear, it has the form f(x) = mx + b

Because it has slope $\frac{1}{3}$ it has the form f(x) = $\frac{1}{3}$x + b (call this Eq.1)

Because (5,4) is on the graph of f, f(5) = 4.

From Eq1. we get f(5) = $\frac{1}{3}$ (5) + b

We have two expression for the same quantity so the two expressions must be equal.

Therefore 4 = $\frac{1}{3}$ (5) + b and it follows that b = $\frac{7}{3}$

The rule for the desired function is f(x) = $\frac{1}{3}$x + $\frac{7}{3}$

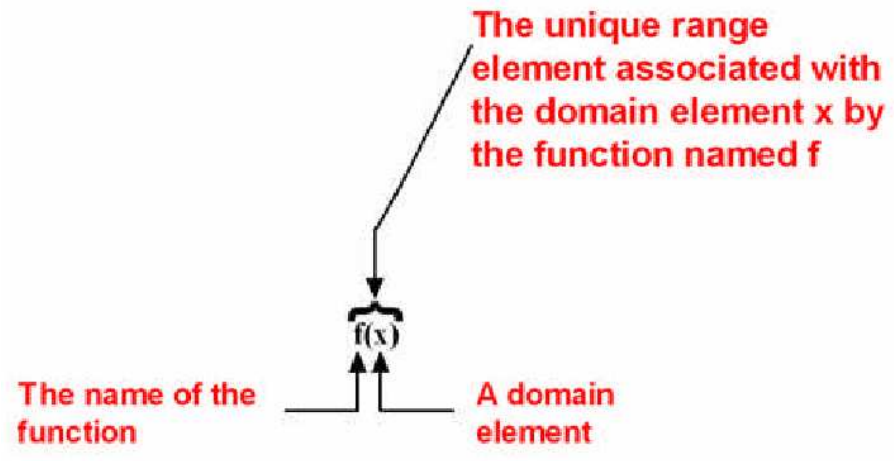
34. Suppose the rule for a function f is $f(x) = \frac{2x-1}{3x^2}$. Calculate the range element associated with –3. (Show your work. Use correct function notation. Use = symbols when appropriate!!!!)

$f(-3) = \frac{2(-3)-1}{3(-3)^2} = \frac{-7}{27}$

35. **(6 pts)** Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.

36. **(4** **pts)**

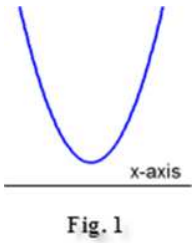
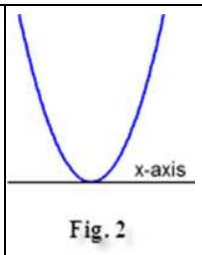
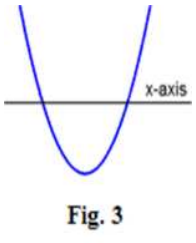
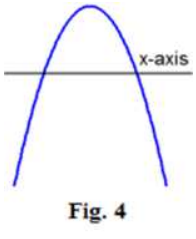
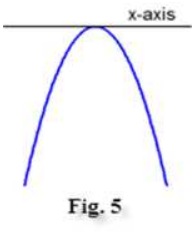
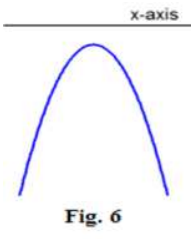


37. **(6 pts)** The orientation of the graph of a quadratic function whose rule is $f(x) = ax^2 + bx + c$ is controlled by the leading coefficient a and the discriminant $b^2 - 4ac$.

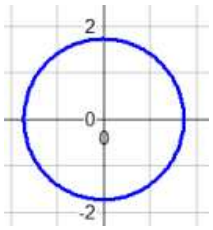
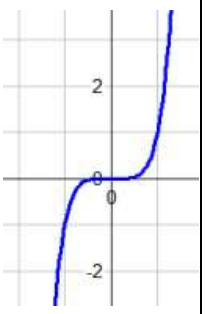
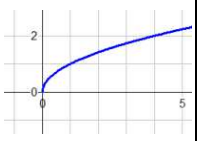
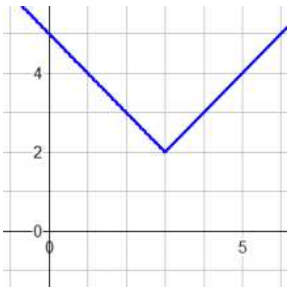
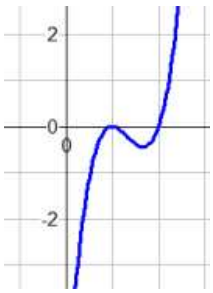
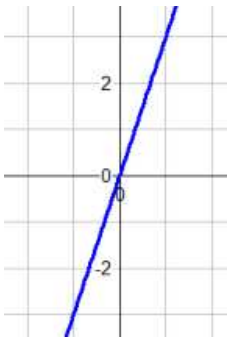
Associate with each graph the correct statement about the leading coefficient and the discriminant.

Fill in the top red blank with the correct one of: **$a < 0$** **$a = 0$** **$a > 0$**

Fill in the bottom red blank with the correct one of: **$b^2 - 4ac < 0$** **$b^2 - 4ac = 0$** **$b^2 - 4ac > 0$**

 Fig. 1	 Fig. 2	 Fig. 3	 Fig. 4	 Fig. 5	 Fig. 6
$a > 0$	$a > 0$	$a > 0$	$a < 0$	$a < 0$	$a < 0$
$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$

38. **(6 pts)** For each of the six graphs below circle the identifying letter if the graph is the graph of a function.

A	B	C	D	E	F
					

39. The point (2, 5) is on the graph of $f(x) = x^2 - 4x + k$. What is the value of k?

Because the point (2,5) is on the graph of f, f(2) = 5

Because the rule for f is $f(x) = x^2 - 4x + k$, $f(2) = 2^2 - 4(2) + k = 4 - 8 + k = -4 + k$

We have two expressions equal to the same quantity, so the two expressions must be equal.

This yields the equation $5 = -4 + k$ which can be solved to conclude that k = 9.

If your paper shows that you do not know simple things like:

- ✓ **the quadrtic formula,**
- ✓ **the distance formula**
- ✓ **the midpoint formula**
- ✓ **the formula for the vertex of a parabola**
- ✓ **the discriminant of a quadratic**
- ✓ **the definition of a function**
- ✓ **function notation**
- ✓ **the form of a linear function**
- ✓ **the form of a quadratic function**

I must conclude that you are not studying.