

NAME: _____ Score _____ /100
Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. (2 pts each)

1. **T** F The norm of a complex number is a real number.
2. T **F** The conjugate of a complex number is a real number.
3. **T** F $\{x|x \in \mathbb{R} \text{ and } 7 \leq x < 12\} = [7, 12)$
4. T **F** $\sqrt{3}$ is a rational number.
5. **T** F If $x - 3$ is added to both sides of an equation, the resulting equation is equivalent to the original.
6. T **F** The reciprocal of $\frac{3}{19}$ is $-\frac{19}{3}$.
7. **T** F $|x| = -5$ is a contradiction.
8. **T** F The set $\{x|x \text{ is an integer less than } 3\}$ contains a negative number.
9. T **F** The solution set for $3x + 2 = 7$ is an interval.
10. T **F** If both sides of an equation are multiplied by $3x - 5$, the resulting equation is equivalent to the original equation.

11. (2 pts) Which of the following are equal to the interval $[4, 7]$. Mark your answers with an X.

- | | | |
|---|---|--|
| <input type="checkbox"/> $\{x 4 < x < 7\}$ | <input type="checkbox"/> $\{x 4 \leq x < 7\}$ | <input type="checkbox"/> $\{4, 5, 6\}$ |
| <input type="checkbox"/> $\{x 4 < x \leq 7\}$ | <input checked="" type="checkbox"/> $\{x 4 \leq x \leq 7\}$ | <input type="checkbox"/> $\{5, 6, 7\}$ |
| <input type="checkbox"/> $[7, 4]$ | <input type="checkbox"/> $(3, 8)$ | <input type="checkbox"/> $\{x x \in \mathbb{N} \text{ and } 4 \leq x \leq 7\}$ |

12. (2 pts) Check each of the following which are equal to the fraction $\frac{x}{y}$.

- | | | |
|---|---|---|
| <input checked="" type="checkbox"/> $\frac{-x}{-y}$ | <input type="checkbox"/> $-\frac{-x}{-y}$ | <input checked="" type="checkbox"/> $-\frac{x}{-y}$ |
| <input type="checkbox"/> $-\frac{x}{y}$ | <input checked="" type="checkbox"/> $-\frac{-x}{y}$ | <input type="checkbox"/> $\frac{-x}{y}$ |
| <input type="checkbox"/> $\frac{x}{-y}$ | <input type="checkbox"/> $\frac{y}{x}$ | |

Fill in each of the blanks to make the statements true. (2 pts each)

13. A binary operation is a **calculation** involving two operands.

14. The multiplicative inverse of a complex number is its **conjugate** divided by its **norm**.
15. A linear equation in one variable is an equation that can be written in the form **$ax + b = 0$** where a and b are real numbers with a not zero.
16. A binary **relation** is a comparison of two operands.
17. If any expression is added to both sides of an equation the resulting equation is **equivalent** to the original equation.
18. Two equations are **equivalent** if they have the same solution sets.
19. An equation is a mathematical statement which contains an = symbol.
20. When both sides of an equation are squared the solution set of the resulting equation **contains** the solution set of the original equation.
21. A number that makes an equation **true** when substituted for the variable is called a solution of the equation.
22. A quadratic equation in one variable is an equation which may be written in the form **$ax^2 + bx + c = 0$** where a, b, and c are real numbers and a is not zero.
23. **(2 pts)** The solution set for an equation in one variable is $\{-2, 0, 3, 4\}$. Sketch the graph of that equation.



24. **(2 pts)** Complete the Quadratic Formula
Quadratic Formula: The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

25. **(2 pts)** Write the compact compound inequality which is equivalent to $|3x - 7| < 9$.

$$-9 < 3x - 7 < 9$$

26. **(2 pts)** If p is a real number and p is not a solution of $4x^3 - 8x + 1 = 0$ and p is not a solution of $4x^3 - 8x + 1 < 0$, then p is a solution of $4x^3 - 8x + 1 > 0$. Why? **Because of The Law of Trichotomy**

27. (5 pts) Solve the linear equation $3x + 5 = 2x + 6$

$$3x + 5 = 2x + 6$$

$$x + 5 = 6$$

$$x = 1$$

28. (5 pts) Use the quadratic formula to solve $x^2 + 5x + 3 = 0$ **No Decimals**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)} = \frac{-5 \pm \sqrt{13}}{2}$$

29. (5 pts) Solve the equation $\sqrt{2x-1} = x-2$

$$\sqrt{2x-1} = x-2$$

$$2x-1 = x^2 - 4x + 4$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

By The Zero Factor Property

$$x-5 = 0 \text{ OR } x-1 = 0$$

$$x = 5 \text{ OR } x = 1$$

Test 5: $\sqrt{2(5)-1} = 5-2$ is TRUE so 5 is a solution

Test 1: $\sqrt{2(1)-1} = 1-2$ is FALSE so 1 is NOT a solution

The solution set for the original equation $\sqrt{2x-1} = x-2$ is $\{5\}$.

30. (5 pts) Solve the inequality $|2x - 1| > 3$

Consider the easier inequality $|2x - 1| < 3$ which is equivalent to

$$-3 < 2x - 1 < 3$$

$$-2 < 2x < 4$$

$$-1 < x < 2$$

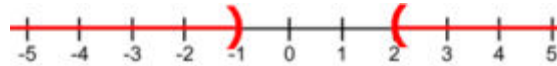
The Solution set for $|2x - 1| < 3$ is the interval $(-1, 2)$.

It is helpful to look at its graph



Therefore the solution set for $|2x - 1| > 3$ is $(-\infty, -1) \cup (2, \infty)$.

and it is instructive to look at its graph



31. (5 pts) Solve $A = 2mw + 2mh + 2wh$ for h .

$$A = 2mw + 2mh + 2wh$$

$$A - 2mw = 2mh + 2wh$$

$$A - 2mw = (2m + 2w)h$$

$$h = \frac{A - 2mw}{2m + 2w}$$

32. (5 pts) (Show your work) $(3 + 5i)(2 - 7i) = 6 - 21i + 10i - 35i^2 = (6 + 35) - 11i = 41 - 11i$

33. (5 pts) The length of a rectangular pool is 6 feet less than twice the width. If the perimeter is 126 feet, what are its dimensions?

$$P = 126$$

$$P = 2x + 2(2x - 6) = 6x - 12$$

We have two expressions for the same quantity.

Therefore by the Transitive Property, they are equal

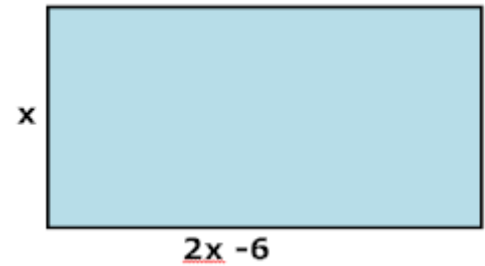
$$6x - 12 = 126$$

$$6x = 138$$

$$x = 23$$

$$\text{Then the length } 2x - 6 \text{ is } 2(23) - 6 = 40$$

The pool is 23 ft. by 40 ft.



34. (2 pts) Complete the following diagram to convert the division problem to an equivalent multiplication problem. (I do not want you to compute the quotient.) No work required.

$$\begin{array}{ccc} (3 + 5i) & \div & (2 - 7i) \\ \downarrow & & \downarrow \end{array}$$

$$(3 + 5i) \cdot \frac{2 + 7i}{2^2 + 7^2}$$

35. (3 pts.) Write $(3, 5]$ using set-builder notation.

$$(3, 5] = \{x \mid 3 < x \leq 5\}$$

36. (3 pts.) Sketch the graph of $(3, 5]$



37. (5 pts.) Consider the complex number $3 - 4i$.

a. What is its real component? **3**

b. What is its complex component? **-4**

c. What is its conjugate? **$3 + 4i$**

d. What is its opposite? **$-3 + 4i$**

e. What is its norm? **$3^2 + 4^2 = 9 + 16 = 25$**

f. What is its multiplicative inverse? **$\frac{3 + 4i}{25}$**

g. What is the product of $3 - 4i$ and its conjugate? **$3^2 + 4^2 = 9 + 16 = 25$**

h. **$(3 - 4i)^2 = -7 - 24i$**

i. **$(3 - 4i) + (2 + i) = 5 - 3i$**

j. **$(3 - 4i) - (2 + i) = 1 - 5i$**