Meramec College Algebra TEST 2 Solution Summer 2010

NAME: _____Score_____/100
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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. (2 pts each for Questions 1 - 20)

1. **T** F The graph of a function is the set of all points of the form (a, f(a)) where a is an element of the domain.
2. T **F** To find the y-intercepts of a graph of a function f we let y = 0 and solve for x.
3. T **F** Composition of functions is a commutative binary operation.
4. **T** F If no horizontal line intersects the graph of a function in more than one point, then the function has an inverse.
5. **T** F A circle is the set of points in a plane that are equidistant from a fixed point called the center.
6. T **F** Every function has an inverse.
7. T **F** f(x) = 3x + 8 is the rule for a quadratic function.
8. When considering the three statements $3x + 5 < 0$, $3x + 5 = 0$, and $3x + 5 > 0$, the statement $3x + 5 = 0$ is called the **boundary equation**.
9. The equation of a circle with radius r and center at the point (h, k) is $(x-h)^2 + (y-k)^2 = r^2$.
10. The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
11. The inverse of a function is the inverse with respect to **composition**.
12. The vertex of the graph of a quadratic function $f(x) = ax^2 + bx + c$ is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
13. The squaring function is the quadratic function f whose rule may be written in the form **$f(x) = x^2$**.
14. A zero of a function f is a domain element k for which **f(k) = 0**.
15. The point (a, b) is on the graph of the function f if and only if b = **f(a)** .
16. The domain of the function whose rule is $f(x) = \frac{x+2}{x-5}$ is all real numbers except **5**.
17. If f and g are functions for which $f \circ g(x) = x$ and $g \circ f(x) = x$, then f and g are **inverses** of each other.
18. A function has an inverse if and only if its graph passes the **horizontal** line test.
19. **Quadratic Formula:** The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
20. The slope of the line through two points (x_1, y_1) and (x_2, y_2) is given by the formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices. ($\frac{1}{2}$ pt. for each choice.)

21. Consider the function whose rule is f(x) = 3x – 7.
    - **a.** f is a linear function
    - **b.** The graph of f is a non-horizontal line.
    - **c.** The graph of f is a non-vertical line.
    - d. The graph of f is a parabola.
    - e. f(2) = 8.
    - f. f does not have an inverse.
    - **g.** f has an inverse.

22. Consider the graph shown in Fig. 1.

Fig. 1

    - **a.** This is the graph of a function.
    - b. The graph passes the horizontal line test.
    - c. This is the graph of a quadratic function.
    - d. This is not the graph of a function.
    - e. This function has an inverse.
    - **f.** This function has at least three real zeros.

23. In function notation
    - a. f(x) is the rule of the function.
    - b. f(x) is a domain element.
    - c. f(x) is the range.
    - **d.** f(x) is a range element.
    - e. f(x) is the function.
    - f. x is the domain.
    - **g.** f is the name of the function.

24. **(7 pts.)** Sketch the graph of the function whose rule is $f(x) = x^2 - 4x - 5$. **Show all computations. Label important points with their coordinates.**

This is a quadratic function so its graph is a parabola which opens up because the leading coefficient is positive.

Note $f(x) = x^2 - 4x - 5 = (x - 5)(x + 1)$

The y-intercept is (0,-5)

The x-intercepts are the real zeros of f which are found by solving the equation resulting from f(x) = 0.

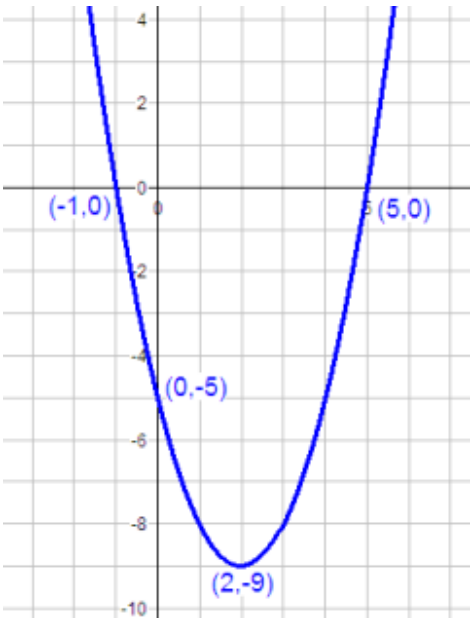
f(x) = 0 if and only if (x – 5)(x + 1) = 0.

By the Zero Factor Property (x – 5)(x + 1) = 0 if and only if x = 5 or -1. Therefore the x-intercepts are (5, 0) and (-1, 0)

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{4}{2}, f\left(\frac{4}{2}\right)\right) = (2, f(2)) = (2, -9)$$

The vertex is

$$\text{Note: } f(2) = (2-5)(2+1) = (-3)(3) = -9$$



25. **(7 pts.)** Find the inverse of the function whose rule is f(x) = 4x – 9.

$f(x) = 4x - 9$

$y = 4x - 9$

$x = 4y - 9$

$x + 9 = 4y$

$y = \frac{1}{4}x + \frac{9}{4}$

The inverse f^{-1} is the function whose rule is $f^{-1}(x) = \frac{1}{4}x + \frac{9}{4}$

26. **(7 pts.)** Consider the two functions f and g whose rules are f(x) = 5x + 2 and $g(x) = x^2$. Show that f and g are NOT inverses of each other.

$f \circ g(x) = f(g(x)) = f(x^2) = 5x^2 + 2 \neq x.$

Because $f \circ g(x) \neq x$, f and g are not inverses or each other.

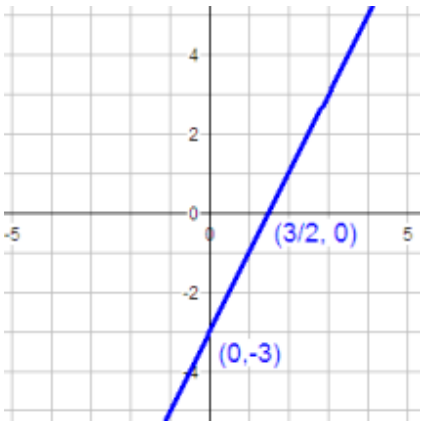
An alternate method is:

$g(f(x)) = g(5x + 2) = (5x + 2)^2 \neq x.$

Because $g \circ f(x) \neq x$, f and g are not inverses or each other.

27. **(7 pts.)** Sketch the graph of the function whose rule is f(x) = 2x – 3. **Show all computations. Label important points with their coordinates.**

f is a linear function so its graph is a non-vertical line.
x and y intercepts are sufficient to determine its graph.
If x = 0, then f(x) = –3.
Therefore the y-intercept is (0, –3) .
If f(x) = 0, then $x = \frac{3}{2}$.
Therefore $\left(\frac{3}{2}, 0\right)$ is the x-intercept.



This list of items labeled a – t are to be used to answer the ten matching questions 1 – 10 below.

a) Law of Trichotomy
b) Transitive Property
c) Distributive Property
c-1) Definition of function
d) Function notation
e) Definition of graph
f) Definition of zero of a function
g) Zero Factor Property
h) Convention when domain is not specified
j) Rule for h
k) Vertical line test
m) Horizontal line test
n) Definition of inverse of a function
p) Definition of linear function
q) Definition of quadratic function
r) Definition of composition
s) Definition of sum of functions
t) Definition of product of functions

MATCHING: For each of the following statements one of the above items is the justification. Enter the correct letter in the blank preceding each of the following numbered statements. Some of the above lettered items might get used more than once and some will not be used. (10 pts)

For the following ten exercises let f, g, and h be functions whose rules are:

f(x) = 4x – 5 $\quad g(x) = \dfrac{x+5}{4} \quad h(x) = \dfrac{x^2-4}{x-5}$

__h__ 1) The domain of h is $(-\infty, 5) \cup (5, \infty)$.

__e__ 2) Because f(1) = –1, the point (1, –1) is on the graph of f.

__s__ 3) $(f+g)(3) = f(3) + g(3)$

__n__ 4) If two functions m and n are inverses of each other, then $m \circ n(4) = 4$

__r__ 5) $f \circ g(x) = f(g(x))$

__b__ 6) If two expressions represent the same quantity, the two expressions are equal.

__b__ 7) Because $g(2) = \dfrac{2+5}{4} = \dfrac{7}{4}$ we conclude that $g(2) = \dfrac{7}{4}$

__f__ 8) $f\left(\dfrac{5}{4}\right) = 0$, therefore $\dfrac{5}{4}$ is a zero of f.

__j__ 9) $h(3a+k) = \dfrac{(3a+k)^2 - 4}{(3a+k) - 5}$

__a__ 10) If a number k is not a solution of the equation 3x + 2 = 5x – 9 and k is not a solution of the inequality 3x + 2 > 5x – 9, then k is a solution of the inequality 3x + 2 < 5x – 9.

(6 pts)Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.

(4 pts)