Meramec College Algebra Chapter 3 Solution TEST 3 Summer 2010

NAME: ______________________________Score_____________________________/100

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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average___________

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 20 are 2 pts each.

1. **T** F The y-intercept of the graph of a quadratic function $f(x) = ax^2 + bx + c$ is $(0, c)$.
2. **T** F A term is a letter, a number, or a product of letters and numbers.
3. **T** F The degree of a polynomial is the degree of the leading term.
4. **T** F The sum of two polynomials is a polynomial.
5. T **F** The sum of two terms is a term.
6. T **F** If the multiplicity of a real zero is an odd number the graph intersects but does not cross the x-axis at that zero.

7. A quadratic function is a function whose rule may be written in the form $f(x) =$ **$ax^2 + bx + c$** where a, b, and c are real numbers and a is not zero.

8. The discriminant of a quadratic function $f(x) = ax^2 + bx + c$ is **b^2-4ac**.

9. The numerical part of a term is called the **coefficient** of the term.

10. If a polynomial contains a term which is strictly numerical, it is called the **constant** term of the polynomial.

11. A **polynomial** is a term or a sum of terms in which all variables have whole number exponents.

12. If a complex number is a zero of a polynomial function f, then its **conjugate** is also a zero of the function f.

13. Division Algorithm: If a and b are natural numbers then there are unique natural numbers q and r such that **$a = bq + r$** with $0 \le r < b$.

14. If f is a polynomial function such that $f(a) < 0$ and $f(b) > 0$, then f has an **x-intercept** (a real zero) between a and b.

15. The graph of a polynomial function is a **smooth continuous** curve with no **sharp** corners.

16. If $\frac{p}{q}$ is a rational zero of a polynomial function with integer coefficients, then the numerator p must be a divisor of the **constant** term and the denominator q must be a divisor of the **leading** coefficient.

17. If the remainder r in the division algorithm is 0, then we say that the polynomial p is **divisible** by the polynomial d.

If f is a polynomial function whose rule is given by

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$

then the following statements are equivalent.

a. k is a real zero of the function f.

18. k is a **solution** of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = 0$.

19. **x – k** is a factor of the polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$.

20. (k, 0) is an **x-intercept** of the graph of the function f.

Questions 21 – 32 are each worth 5 pts.

21. Find the vertex of the graph of the function whose rule is $f(x) = x^2 + 5x + 6$. **Show the steps**

The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(-\frac{5}{2}, f\left(-\frac{5}{2}\right)\right) = \left(-\frac{5}{2}, -\frac{1}{4}\right)$

Note $f(x)=x^2+5x+6=(x+2)(x+3)$

So that $f\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}+2\right)\left(-\frac{5}{2}+3\right)=\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)=-\frac{1}{4}$

22. What are the x-intercepts of the graph of the function whose rule is $f(x) = (x + 3)(x – 5)(x – 2)$?

The x-intercepts are (–3, 0), (5, 0), and (2, 0)

23. Consider the polynomial function whose rule is $f(x) = –43x^5 + 281x^3 – 97x^2 + 2x –302$. Determine the end behavior of the graph of f by completing the following:

As $x \to +\infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to +\infty$

24. Consider the polynomial function whose rule is $f(x) = 5x^4 + 7x^2 – x + 9$. Determine the possible rational zeros of f by completing the following:

$p \in \{\pm 1, \pm 3, \pm 9\}$

$q \in \{\pm 1, \pm 5\}$

$\frac{p}{q} \in \left\{\pm 1, \pm 3, \pm 9, \pm\frac{1}{5}, \pm\frac{3}{5}, \pm\frac{9}{5}\right\}$

25. Which of the following are rules for polynomial functions? Identify your choices by placing an X in the box preceding the option.

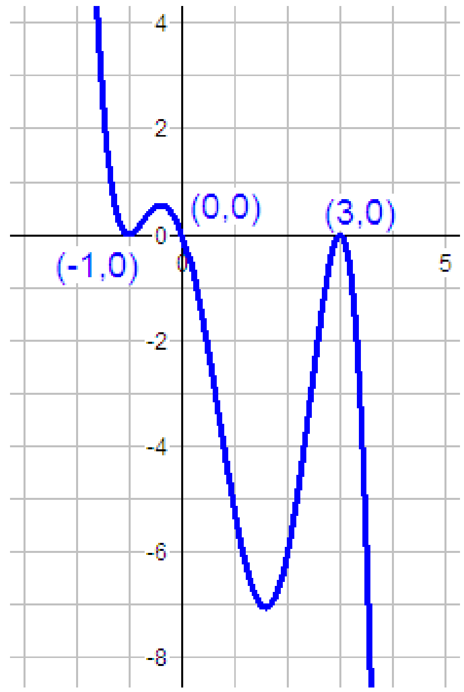
☐ **$f(x) = 2x - 4$** ☐ $f(x) = 3x^{-5} - 8x^{\frac{1}{2}}$ ☐ $f(x) = \dfrac{3x^2 + 2x^5 + 4x - 2}{2}$

☐ $f(x) = \dfrac{3x^4 - 2x^5 + 4x - 2}{2x^3 + 5}$ ☐ **$f(x) = (2x - 4)(x^2+4)$**

26. Use the Intermediate Value Theorem to prove that the graph of the function whose rule is $f(x) = x^5 - x^3 - 1$ has an x-intercept between 1 and 2. **Both calculations and words of explanation are required.**

$f(1) = 1^5 - 1^3 - 1 = 1 - 1 - 1 < 0$ and $f(2) = 25 - 23 - 1 = 64 - 8 - 1 > 0$

Therefore, by the Intermediate Value Theorem, there is an x-intercept between 1 and 2.



27. An analysis of a function f reveals the following facts.
   a. f is a polynomial function of degree 5.
   b. The real zeros of f are -1, 0, and 3.
   c. The multiplicity of -1 is 2.
   d. The multiplicity of 3 is 2.
   e. As $x \to +\infty$, $f(x) \to -\infty$
      As $x \to -\infty$, $f(x) \to +\infty$

Sketch the graph of f.

28. **Circle all the words which can be used to correctly complete the sentences.**

a. $f(x) = 7$ is the rule for a (**constant** **linear** quadratic identity **polynomial**) function.

b. $f(x) = x^2 + 2x + 1$ is the rule for a (constant linear **quadratic** identity **polynomial**) function.

c. $f(x) = x^5 + 5x + 6$ is the rule for a (constant linear quadratic identity **polynomial**) function.

d. $f(x) = x + 4$ is the rule for a (constant **linear** quadratic identity **polynomial**) function.

e. $f(x) = x$ is the rule for a (constant **linear** quadratic **identity** **polynomial**) function.

29. Consider the polynomial function f whose rule is $f(x) = (x+1)^5(x^2 - 2)^2$.

a. What is the degree of f? **9**

b. What is the leading coefficient of f? **x^9**

c. What is the constant term of f? **4**

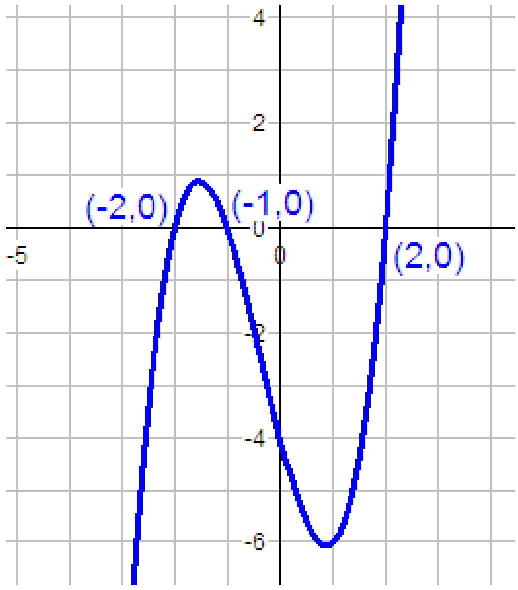
d. What are the zeros of f? $-1,\ \sqrt{2},\ -\sqrt{2}$

e. How many times does the graph of f cross the x-axis? **Once**

30. Consider the function whose rule is $f(x) = x^3 + x^2 - 4x - 4$. The zeros of f are –2, –1, and 2. Sketch the graph of f.

As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to -\infty$



31. Consider the function whose rule is $f(x) = x^3 + x^2 - 4x - 4$. The zeros of f are –2, –1, and 2.
Factor the polynomial $x^3 + x^2 - 4x - 4$

$x^3 + x^2 - 4x - 4 = (x+2)(x+1)(x-2)$

32. Perform the indicated long division:

$$\begin{array}{r} 6x^2 + 3x - 1 \\ 3x^2 + 1 \overline{)\, 18x^4 + 9x^3 + 3x^2} \\ \underline{18x^4 \qquad + 6x^2} \\ 9x^3 - 3x^2 \\ \underline{9x^3 \qquad + 3x} \\ -3x^2 - 3x \\ \underline{-3x^2 \qquad - 1} \\ -3x + 1 \end{array}$$