General Comments Concerning Solutions

1. Remember that when working math exercises your goal is not to “get the answer”.

a) Your primary goal is to determine if you understand the concepts, which you have been studying, well enough to use them to solve problems.
b) A secondary goal is to further learn and understand the theory and process used to answer the question.
c) A third goal is to communicate to yourself and your instructor that you understand the theory and processes.
d) Proper communication is a by-product of the three goals.

2. In the attached solutions I have present two styles distinguished by their titles.

a) Solution/Reasoning
b) Solution

In those cases where Solution/Reasoning is presented, I write most of the important thought process that must be part of the solution process. When you are learning a new process, it is a good idea to write out all of the reasoning as I have done when presenting Solution/Reasoning. By writing these particular details, you force yourself to concentrate on the critical mathematics facts which you can carry forward to other situations. After you become more practiced at a particular process, you may revert to writing a more abbreviated form as presented in the more sparse Solution presentations used in these solutions. However, even when writing the simpler form, your mind should concentrate on the underlying mathematics details and that should guide you through the process. The process should always be based on mathematics not on a memorization of steps as presented by Teacher Fritz.

3. This particular environment – solving equations involving radicals – demonstrates some very important basic ideas. These ideas have far reaching consequences.

a) Solving an equation means to find all numbers which make the equation true.
b) The process used must find solutions.
c) The process used must insure that all solutions have been discovered.
d) The process must provide a means of recovery when it has a flaw.
e) The process might yield a list of possible solutions which must be tested.
f) Every step in a process must be justified by some mathematics principle.

Equations with Variables in Radicals

Exercises Solutions

Problem 1: Solve the equation $\sqrt{2x+9} = x+3$
Solution/Reasoning: Square both sides of the original equation to obtain the equation $2x + 9 = x^2 + 6x + 9$.
Note: This equation might not be equivalent to the original equation.
However, the solution set of this equation contains the solution set of the first equation.
Add $-2x - 9$ to both sides of the second equation to obtain $x^2 + 4x = 0$.
Factorization yields $x(x + 4) = 0$ and then The Zero Factor Property yields the two equations $x = 0$ OR $x + 4 = 0$. The solution sets to these two equations are {0} OR {–4}.
The solution set of the second equation $2x + 9 = x^2 + 6x + 9$ is therefore $\{0\} \cup \{-4\} = \{0, -4\}$.
Every solution of the original equation is contained in the set {0, –4}, but some of the elements of the set {0, –4} might not be solutions of the original equation. Therefore we must test each of these possible solutions.
Test 0: $\sqrt{2(0)+9} = 0+3$ is TRUE.
Therefore 0 is a solution of the original equation.
Test –4: $\sqrt{2(-4)+9} = -4+3$

$1 = -1$ is FALSE

Therefore –4 is not a solution of the original equation.
Therefore the solution set for the original equation is the set {0}.
Observe that $\{0\} \subset \{0, -4\}$.

An Alternate Way to Write the Solution Process

$\sqrt{2x+9} = x+3$	square both sides
$2x + 9 = x^2 + 6x + 9$	add $-2x - 9$ to both sides
$x^2 + 4x = 0$	factor
$x(x + 4) = 0$	

By The Zero Factor Property

$x = 0$ OR $x + 4 = 0$

$x = 0$ OR $x = -4$

The solution set for the equation $2x + 9 = x^2 + 6x + 9$ is {0, –4}
The solution set for the original equation is a subset of {0, –4}

Test 0:	**Test –4:**
$\sqrt{2(0)+9} = 0+3$ is TRUE. Therefore 0 is a solution of the original equation.	$\sqrt{2(-4)+9} = -4+3$ $-1 = 1$ is FALSE Therefore –4 is not a solution of the original equation.

It follows that the solution set for the original equation is the set {0}.

Problem 2: Solve the equation $\sqrt{10x+5}-1=2x$

Solution:

$\sqrt{10x+5}-1=2x$	add 1 to both sides
$\sqrt{10x+5}=2x+1$	square both sides
$10x + 5 = 4x^2 + 4x + 1$	add –10x – 5 to both sides
$4x^2 – 6x – 4 = 0$	multiply both sides by $\frac{1}{2}$
$2x^2 – 3x – 2 = 0$	factor

(2x + 1)(x – 2) = 0

By The Zero Factor Property

2x + 1 = 0 OR x – 2 = 0

$x=-\frac{1}{2}$ OR $x=2$

The solution set for $10x + 5 = 4x^2 + 4x + 1$ is $\left\{-\frac{1}{2},2\right\}$

The solution set for the original equation is a subset of $\left\{-\frac{1}{2},2\right\}$

Test $-\frac{1}{2}$**:** $\sqrt{10\left(\frac{-1}{2}\right)+5}-1=2\left(-\frac{1}{2}\right)$ is TRUE Therefore $-\frac{1}{2}$ is a solution of the original equation.	**Test 2:** $\sqrt{10(2)+5}-1=2(2)$ is TRUE Therefore 2 is a solution of the original equation.

The solution set for the original equation is $\left\{-\frac{1}{2},2\right\}$

Problem 3: Solve the equation $\sqrt{5x+1}-\sqrt{3x}=1$
Solution:

$\sqrt{5x+1}-\sqrt{3x}=1$	add $\sqrt{3x}$ to both sides
$\sqrt{5x+1}=\sqrt{3x}+1$	square both sides
$5x+1=3x+2\sqrt{3x}+1$	add – 3x – 1 to both sides
$2x=2\sqrt{3x}$	multiply both sides by $\frac{1}{2}$
$x=\sqrt{3x}$	square both sides
$x^2 = 3x$	add – 3x to both sides
$x^2 - 3x = 0$	factor
$x(x - 3) = 0$	

By The Zero Factor Property

x = 0 OR x – 3 = 0

x = 0 OR x = 3

The solution set for $x^2 = 3x$ is {0, 3}.

The solution set for the original equation is a subset of {0, 3}.

Test 0**:** $\sqrt{5(0)+1}-\sqrt{3(0)}=1$ is TRUE Therefore 0 is a solution of the original equation.	**Test 3:** $\sqrt{5(3)+1}-\sqrt{3(3)}=1$ is TRUE Therefore 3 is a solution of the original equation.

The solution set for the original equation is {0, 3}.

Problem 4: Solve the equation $\sqrt{3x+1} = 1 + \sqrt{x+4}$

Solution:

$\sqrt{3x+1} = 1 + \sqrt{x+4}$ square both sides

$3x + 1 = 1 + 2\sqrt{x+4} + x + 4$ add $-x - 5$ to both sides

$2x - 4 = 2\sqrt{x+4}$ multiply both sides by $\frac{1}{2}$

$x - 2 = \sqrt{x+4}$ square both sides

$x^2 - 4x + 4 = x + 4$ add $- x - 4$ to both sides

$x^2 - 5x = 0$ factor

$x(x - 5) = 0$

By The Zero Factor Property

$x = 0$ OR $x - 5 = 0$

$x = 0$ OR $x = 5$

The solution set for $x^2 = 3x$ is {0, 5}.

The solution set for the original equation is a subset of {0, 5}.

Test 0: $\sqrt{3(0)+1} = 1 + \sqrt{0+4}$ is FALSE Therefore 0 is not a solution of the original equation.	**Test 5:** $\sqrt{3(5)+1} = 1 + \sqrt{5+4}$ is TRUE Therefore 5 is a solution of the original equation.

The solution set for the original equation is {5}.

Problem 5: Solve the equation $\sqrt{2x+5}-\sqrt{x-1}=\sqrt{x+2}$

Solution:

$\sqrt{2x+5}-\sqrt{x-1}=\sqrt{x+2}$ square both sides

$(2x+5)-2\sqrt{2x+5}\sqrt{x-1}+(x-1)=x+2$ add – 2x – 5 to both sides

$-2\sqrt{2x+5}\sqrt{x-1}+(x-1)=-x-3$ add – x + 1 to both sides

$-2\sqrt{2x+5}\sqrt{x-1}=-2x-2$ multiply both sides by $-\frac{1}{2}$

$\sqrt{2x+5}\sqrt{x-1}=x+1$ square both sides

$(2x + 5)(x – 1) = x^2 + 2x + 1$ multiply

$2x^2 + 3x – 5 = x^2 + 2x + 1$ add – $(x^2 + 2x + 1)$ to both sides

$x^2 + x – 6 = 0$ factor

$(x + 3)(x – 2) = 0$

By The Zero Factor Property

$x + 3 = 0$ OR $x – 2 = 0$

$x = –3$ OR $x = 2$

The solution set for $(2x + 5)(x – 1) = x^2 + 2x + 1$ is {–3, 2}.

The solution set for the original equation is a subset of {–3, 2}.

Test –3: $\sqrt{2(-3)+5}-\sqrt{-3-1}=\sqrt{-3+2}$ $\sqrt{-1}-\sqrt{-4}=\sqrt{-1}$ The last line shows that –3 is not in the domain of the original equation. (Because these square roots are not real numbers.) Therefore –3 is not a solution of the original equation.	**Test 2:** $\sqrt{2(2)+5}-\sqrt{2-1}=\sqrt{2+2}$ $\sqrt{9}-\sqrt{1}=\sqrt{4}$ is TRUE Therefore 2 is a solution of the original equation.

The solution set for the original equation is {2}.

Problem 6: Solve the equation $x = \sqrt{-5x-6}$

Solution/Reasoning: Square both sides of the original equation to obtain the equation $x^2 = -5x - 6$.

Note: This equation might not be equivalent to the original equation.

However, the solution set of this equation contains the solution set of the first equation.

Add 5x + 6 to both sides of the second equation to obtain $x^2 + 5x + 6 = 0$.

Factorization yields (x + 2)(x + 3) = 0 and then The Zero Factor Property yields the two equations x + 2 = 0 OR x + 3 = 0.

The solution sets to these two equations are {-2} OR {-3}.

The solution set of the second equation $x^2 = -5x - 6$ is therefore $\{-2\} \cup \{-3\} = \{-2, -3\}$.

Every solution of the original equation is contained in the set {–2, –3}, but some of the elements of the set {–2, –3} might not be solutions of the original equation.

Therefore we must test each of these possible solutions.

Test –2: $-2 = \sqrt{-5(-2)-6} = \sqrt{4} = 2$ is FALSE

Therefore –2 is not a solution of the original equation.

Test –3: $-3 = \sqrt{-5(-3)-6} = \sqrt{9} = 3$ is FALSE

Therefore –3 is not a solution of the original equation.

Neither of the two possible solutions are solutions of the original equation.

Therefore the solution set for the original equation is the empty set $\varnothing$

Observe that $\varnothing \subset \{-2, -3\}$

An Alternate Way to Write the Solution Process

$x = \sqrt{-5x-6}$	square both sides
$x^2 = -5x - 6$	add 5x + 6 to both sides
$x^2 + 5x + 6 = 0 = 0$	factor

(x + 3)(x + 2) = 0

By The Zero Factor Property

x + 3 = 0 OR x + 2 = 0

x = – 3 OR x = – 2

The solution set for the equation $x^2 = -5x - 6$ is {– 3, – 2}

The solution set for the original equation is a subset of {– 3, – 2}

Test – 2:	**Test – 3:**
$-2 = \sqrt{-5(-2)-6} = \sqrt{4} = 2$ is False Therefore – 2 is not a solution of the original equation.	$-3 = \sqrt{-5(-3)-6} = \sqrt{9} = 3$ is FALSE Therefore – 3 is not a solution of the original equation.

It follows that the solution set for the original equation is the empty set $\varnothing$.

Problem 7: Solve the equation $x = \sqrt{5x-6}$

Solution/Reasoning: Square both sides of the original equation to obtain the equation $x^2 = 5x - 6$.

Note: This equation might not be equivalent to the original equation.

However, the solution set of this equation contains the solution set of the first equation.

Add $-5x + 6$ to both sides of the second equation to obtain $x^2 - 5x + 6 = 0$.

Factorization yields $(x - 2)(x - 3) = 0$ and then The Zero Factor Property yields the two equations $x - 2 = 0$ OR $x - 3 = 0$.

The solution sets to these two equations are {2} OR {3} respectively.

The solution set of the second equation $x^2 - 5x + 6 = 0$ is therefore $\{2\} \cup \{3\} = \{2, 3\}$.

Every solution of the original equation is contained in the set {2, 3}, but some of the elements of the set {2, 3} might not be solutions of the original equation. Therefore we must test each of these possible solutions.

Test 2: $2 = \sqrt{5(2)-6}$ is TRUE

Therefore 2 is a solution of the original equation.

Test 3: $3 = \sqrt{5(3)-6}$ is TRUE

Therefore 3 is a solution of the original equation.

We conclude the solution set for the original equation is the set {2, 3}.

Observe that $\{2, 3\} \subset \{2, 3\}$.

An Alternate Way to Write the Solution Process

$x = \sqrt{5x-6}$	square both sides
$x^2 = 5x - 6$	add $-5x + 6$ to both sides
$x^2 - 5x + 6 = 0 = 0$	factor

$(x - 3)(x - 2) = 0$

By The Zero Factor Property

$x - 3 = 0$ OR $x - 2 = 0$

$x = 3$ OR $x = 2$

The solution set for the equation $x^2 = 5x - 6$ is {3, 2}

The solution set for the original equation is a subset of {3, 2}

Test 2:	**Test 3:**
$2 = \sqrt{5(2)-6}$ is TRUE Therefore 2 is a solution of the original equation.	$3 = \sqrt{5(3)-6}$ is TRUE Therefore 3 is a solution of the original equation.

It follows that the solution set for the original equation is the set {2, 3}.