

## Function Notation Exercises

### Part I:

- Write each of the following mathematical statements using normal English prose.
- Write what each of these statements tells you about the graph of the function.

**Example:** The statement  $f(3) = 7$  should be written as:

- The unique range element associated with the domain element 3 by the function  $f$  is 7.
- The point  $(3, 7)$  is on the graph of  $f$ .

- |                    |                             |                         |
|--------------------|-----------------------------|-------------------------|
| 1) $f(2) = 8$      | 2) $f(5) = (5)(67)$         | 3) $f(\pi) = \sqrt{23}$ |
| 4) $f(x) = 3x + 1$ | 5) $f(x) = 2^x + 5^x$       | 6) $f(x) = \pi^x$       |
| 7) $f(x) = x^\pi$  | 8) $f \circ g(x) = f(g(x))$ | 9) $f \circ g(2) = 5$   |
| 10) $f(7) > 48$    | 11) $f(2) < 5^2$            | 12) $f(3) > g(x)$       |

### Part II:

- Write each of the following mathematical statements using correct mathematics notation.

**Example:** The statement; "The range element associated with the domain element 4 by the function  $f$  is 7" should be written as:  $f(4) = 7$ .

- The range element associated with the domain element 8 by the function  $f$  is 11.
- 45 is the range element associated with the domain element 9 by the function  $g$ .
- The rule for the function  $f$  is: The range element associated with a domain element is the domain element raised to the 11<sup>th</sup> power.
- The range element associated with the domain element 14 is less than the range element associated with the domain element  $\frac{3}{\sqrt{7}}$  by the same function  $f$ .
- Two functions  $f$  and  $g$  have the same domains and for each domain element, the range element associated with it by the function  $g$  is greater than the range element associated with it by the function  $f$ .
- The point  $(3, 5)$  is on the graph of the function  $h$ .
- The point  $\left(\frac{3}{4}, \frac{2\pi}{3}\right)$  is on the graph of the function  $jim$ .
- 86 is a zero of the function  $k$ .
- The function  $f$  is a linear function.
- The function  $f$  is a quadratic function.

## Function Notation Exercises **Solutions**

### Part I:

- a) Write each of the following mathematical statements using normal English prose.
- b) Write what each of these statements tells you about the graph of the function.

**Example:** The statement  $f(3) = 7$  should be written as:

- a) The unique range element associated with the domain element 3 by the function  $f$  is 7.
- b) The point  $(3, 7)$  is on the graph of  $f$ .

1)  $f(2) = 8$

- a) The unique range element associated with 2 by the function  $f$  is equal to 8.
- b) The point  $(2, 8)$  is on the graph of the function  $f$ .

2)  $f(5) = (5)(67)$

- a) The unique range element associated with 5 by the function  $f$  is equal to 5 times 67.
- b) The point  $(5, (5)(67))$  is on the graph of the function  $f$ . Note that this point may also be specified as  $(5, 335)$

3)  $f(\pi) = \sqrt{23}$

- a) The unique range element associated with  $\pi$  by the function  $f$  is equal to  $\sqrt{23}$ .
- b) The point  $(\pi, \sqrt{23})$  is on the graph of the function  $f$ . Note that the second coordinate of this point is an irrational number between 4 and 5 and is closer to 5 than to 4.

4)  $f(x) = 3x + 1$

- a) The unique range element associated with  $x$  by the function  $f$  is equal to  $3x + 1$ . Note that this is in fact a general rule for the function  $f$ . That rule can be stated as follows: The unique range element associated with a domain element by the function  $f$  is one plus three times the domain element.
- b) The point  $(x, 3x + 1)$  is on the graph of the function  $f$ .

5)  $f(x) = 2^x + 5^x$

- a) The unique range element associated with  $x$  by the function  $f$  is equal to 2 raised to the  $x^{\text{th}}$  power plus 5 raised to the  $x^{\text{th}}$  power. Note that this is in fact a general rule for the function  $f$ . Later in this course it will become clear that this is an example of an exponential function.
- b) The point  $(x, 2^x + 5^x)$  is on the graph of the function  $f$ . The graph of  $f$  consists of all such points.

6)  $f(x) = \pi^x$

- a) The unique range element associated with  $x$  by the function  $f$  is equal to  $\pi$  raised to the  $x^{\text{th}}$  power. Note that this is in fact a general rule for the function  $f$ . Later in this course it will become clear that this is an example of an exponential function.
- b) The point  $(x, \pi^x)$  is on the graph of the function  $f$ . The graph of  $f$  consists of all such points.

7)  $f(x) = x^\pi$

- a) The unique range element associated with  $x$  by the function  $f$  is equal to  $x$  raised to the  $\pi^{\text{th}}$  power. Note that this is in fact a general rule for the function  $f$ . This is not an exponential function because the exponent is not variable.
- b) The point  $(x, x^\pi)$  is on the graph of the function  $f$ . The graph of  $f$  consists of all such points.

8)  $f \circ g(x) = f(g(x))$

- a) The unique range element associated with  $x$  by the function  $f \circ g$  is equal to the unique range element associated with the domain element  $g(x)$  by the function  $f$ . Note however, that although  $g(x)$  is a domain element for the function  $f$ , with respect to the function  $g$  it is the unique range element associated with the domain element  $x$  by the function  $g$ . It is almost impossible to write this in a meaningful way with ordinary English, but here is an attempt.

The unique range element associated with  $x$  by the function  $f \circ g$  is equal to the unique range element associated, by  $f$ , with the unique range element associated with the element  $x$  of the domain of  $g$  by the function  $g$ . This references the standard diagram for composition of function (later in this course).

- b) Has no graphical interpretation.

9)  $f \circ g(2) = 5$

- a) The unique range element associated with 2 by the function  $f \circ g$  is equal to 5.
- b) The point  $(2, 5)$  is on the graph of the function  $f \circ g$ .

10)  $f(7) > 48$

- a) The unique range element associated with 7 by the function  $f$  is greater than 48.
- b) The point  $(7, f(7))$  is on the graph of the function  $f$  and is above the horizontal line  $y = 48$ .

11)  $f(2) < 5^2$

- a) The unique range element associated with 2 by the function  $f$  is greater than 25.
- b) The point  $(2, f(2))$  is on the graph of the function  $f$  and is above the horizontal line  $y = 25$ .

12)  $f(3) > g(x)$

- a) The unique range element associated with 3 by the function  $f$  is greater than unique range elements associated with domain elements by the function  $g$ .
- b) The point  $(3, f(3))$  is on the graph of the function  $f$  and is above every point on the graph of  $g$ .

## Part II:

- a) Write each of the following mathematical statements using correct mathematics notation.

**Example:** The statement; "The range element associated with the domain element 4 by the function  $f$  is 7" should be written as:  $f(4) = 7$ .

- 1) The range element associated with the domain element 8 by the function  $f$  is 11.

This statement can be written as  $f(8) = 11$ .

- 2) 45 is the range element associated with the domain element 9 by the function  $g$ .

This statement can be written as  $g(9) = 45$ .

- 3) The rule for the function  $f$  is: The range element associated with a domain element is the domain element raised to the 11<sup>th</sup> power.

The rule for the function  $f$  is:  $f(x) = x^{11}$

- 4) The range element associated with the domain element 14 is less than the range element associated with the domain element  $\frac{3}{\sqrt{7}}$  by the same function  $f$ .

This statement can be written as  $f(14) < f\left(\frac{3}{\sqrt{7}}\right)$ .

- 5) Two functions  $f$  and  $g$  have the same domains and for each domain element, the range element associated with it by the function  $g$  is greater than the range element associated with it by the function  $f$ .

Functions  $f$  and  $g$  have the same domains and for every element of this common domain  $g(x) > f(x)$ .

- 6) The point  $(3, 5)$  is on the graph of the function  $h$ .

This statement can be written as  $h(3) = 5$ .

- 7) The point  $\left(\frac{3}{4}, \frac{2\pi}{3}\right)$  is on the graph of the function  $jim$ .

This statement can be written as  $jim\left(\frac{3}{4}\right) = \frac{2\pi}{3}$

- 8) 86 is a zero of the function  $k$ .

This statement can be written as  $k(86) = 0$ .

- 9) The function  $f$  is a linear function.

The statement may be written as: The rule for the function  $f$  has the form  $f(x) = mx + b$  where  $m$  and  $b$  are real numbers.

- 10) The function  $f$  is a quadratic function.

The statement may be written as: The rule for the function  $f$  has the form  $f(x) = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ .