Rational Expressions and Rational Equations
Exercises and Solutions

1. Simplify $\dfrac{15x^2y}{5xy^2}$

2. Simplify $\dfrac{x-7}{7-x}$

3. Simplify $\dfrac{x^2-4}{x^2-x-2}$

4. Multiply and simplify $\left(\dfrac{2x+6}{x+3}\right)\left(\dfrac{3}{4x}\right)$

5. Multiply and simplify $16x\left(\dfrac{3x+8}{4x}\right)$

6. Divide and simplify $12x \div \left(\dfrac{16x^2}{x+4}\right)$

7. Add $\dfrac{5x}{7}+\dfrac{9x}{7}$

8. Add $\dfrac{3x}{x^2-2x+1}+\dfrac{6-x}{x^2-2x+1}$

9. Expand the fraction $\dfrac{2y}{x}$ to a fraction with a denominator of x(x + 3)

10. Add $\dfrac{2}{5x+25}+\dfrac{4}{x^2-25}$

11. Subtract $\dfrac{x-1}{9x}-\dfrac{x-2}{x^3}$

12. Simplify the complex fraction $\dfrac{\frac{1}{y}-\frac{5}{2}}{\frac{3}{y}}$

13. Simplify the complex fraction $\dfrac{\frac{2x-8}{15}}{\frac{3x-12}{35x}}$

14. Solve the equation $\dfrac{2}{3}=\dfrac{1}{2}+\dfrac{x}{6}$

15. Solve the equation $\dfrac{x}{4}-\dfrac{4}{x}=0$

16. Solve the equation $\dfrac{3}{x-2}+1=\dfrac{3}{x-2}$

17. Solve the equation $\dfrac{x-4}{x-3}+\dfrac{x-2}{x-3}=x-3$

18. Solve $I=\dfrac{E}{R+r}$ for r

1. Simplify $\frac{15x^2y}{5xy^2} = \frac{(5)(3)(x)(x)(y)}{(5)(x)(y)(y)} = \left(\frac{(5)(x)(y)}{(5)(x)(y)}\right)\left(\frac{3x}{y}\right) = \frac{3x}{y}$

2. Simplify $\frac{x-7}{7-x} = \frac{(-1)(7-x)}{7-x} = (-1)\left(\frac{7-x}{7-x}\right) = -1$

3. Simplify $\frac{x^2-4}{x^2-x-2} = \frac{(x-2)(x+2)}{(x-2)(x+1)} = \left(\frac{x-2}{x-2}\right)\left(\frac{x+2}{x+1}\right) = \frac{x+2}{x+1}$

4. Multiply and simplify $\left(\frac{2x+6}{x+3}\right)\left(\frac{3}{4x}\right) = \frac{2(x+3)(3)}{(x+3)4x} = \frac{2(x+3)}{2(x+3)}\left(\frac{3}{2x}\right) = \frac{3}{2x}$

5. Multiply and simplify $16x\left(\frac{3x+8}{4x}\right) = \left(\frac{4x}{4x}\right)\frac{(4)(3x+8)}{1} = (4)(3x+8) = 12x+32$

6. Divide and simplify $12x \div \left(\frac{16x^2}{x+4}\right) = \left(\frac{12x}{1}\right)\left(\frac{x+4}{16x^2}\right) = \left(\frac{4x}{4x}\right)\left(\frac{3}{4x}\right)\left(\frac{x+4}{1}\right) = \frac{3x+12}{4x}$

7. Add $\frac{5x}{7} + \frac{9x}{7} = \frac{5x+9x}{7} = \frac{14x}{7} = 2x$

8. Add $\frac{3x}{x^2-2x+1} + \frac{6-x}{x^2-2x+1} = \frac{3x+(6-x)}{x^2-2x+1} = \frac{2x+6}{x^2-2x+1}$

9. Expand the fraction $\frac{2y}{x}$ to a fraction with a denominator of x(x + 3)

$$\frac{2y}{x} = \left(\frac{2y}{x}\right)\left(\frac{x+3}{x+3}\right) = \frac{(2y)(x+3)}{(x)(x+3)} = \frac{2xy+6y}{(x)(x+3)}$$

10. Add $\frac{2}{5x+25} + \frac{4}{x^2-25} = \frac{2}{5(x+5)} + \frac{4}{(x+5)(x-5)} =$

$$\frac{(2)(x-5)}{(5)(x+5)(x-5)} + \frac{(4)(5)}{(5)(x+5)(x-5)} = \frac{(2)(x-5)+(4)(5)}{(5)(x+5)(x-5)}$$

$$= \frac{2x+10}{(5)(x+5)(x-5)} = \frac{2(x+5)}{(5)(x+5)(x-5)} = \left(\frac{2}{(5)(x-5)}\right)\left(\frac{(x+5)}{(x+5)}\right) = \frac{2}{5(x-5)}$$

11. Subtract $\frac{x-1}{9x} - \frac{x-2}{x^3} = \frac{x-1}{9x} + \left(-\frac{x-2}{x^3}\right) = \frac{x-1}{9x} + \left(\frac{2-x}{x^3}\right) = \left(\frac{x-1}{9x}\right)\left(\frac{x^2}{x^2}\right) + \left(\frac{2-x}{x^3}\right)\left(\frac{9}{9}\right)$

$$= \frac{x^3-x^2}{9x^3} + \frac{18-9x}{9x^3} = \frac{\left(x^3-x^2\right)+(-9x+18)}{9x^3} = \frac{x^3-x^2-9x+18}{9x^3}$$

12. Simplify the complex fraction $\dfrac{\dfrac{1}{y}-\dfrac{5}{2}}{\dfrac{3}{y}}$

$$\frac{\frac{1}{y}-\frac{5}{2}}{\frac{3}{y}}=\frac{\left(\frac{1}{y}\right)\left(\frac{2}{2}\right)-\left(\frac{5}{2}\right)\left(\frac{y}{y}\right)}{\frac{3}{y}}=\frac{\frac{2}{2y}-\frac{5y}{2y}}{\frac{3}{y}}=\frac{\frac{2-5y}{2y}}{\frac{3}{y}}=\left(\frac{2-5y}{2y}\right)\left(\frac{y}{3}\right)=\left(\frac{2-5y}{(2)(3)}\right)\left(\frac{\cancel{y}}{\cancel{y}}\right)=\frac{2-5y}{6}$$

13. Simplify the complex fraction $\dfrac{\dfrac{2x-8}{15}}{\dfrac{3x-12}{35x}}$

$$\frac{\frac{2x-8}{15}}{\frac{3x-12}{35x}}=\left(\frac{2x-8}{15}\right)\left(\frac{35x}{3x-12}\right)=\left(\frac{2(x-4)}{(3)(5)}\right)\left(\frac{(5)(7)x}{3(x-4)}\right)=\left(\frac{\cancel{5(x-4)}}{\cancel{5(x-4)}}\right)\left(\frac{(2)(7)x}{(3)(3)}\right)=\frac{14x}{9}$$

14. Solve the equation $\dfrac{2}{3}=\dfrac{1}{2}+\dfrac{x}{6}$

Multiply both sides of the equation by 6 to obtain the equivalent equation 4 = 3 + x.

Add -3 to both sides of the equation to obtain the simplest equation x = 1

The solution set is {1}

15. Solve the equation $\dfrac{x}{4}-\dfrac{4}{x}=0$

Multiply both sides of the equation by 4x to obtain the equation x^2 -16 = 0.

Note this equation need not be equivalent to the original.

Factor x^2 -16 to obtain the equation (x - 4)(x + 4) = 0.

By the Zero Factor Property then x = 4 or x = -4.

Observe that neither of these cause a zero in the denominator and both are therefore solutions.

The solution set for the original equation is {4, -4}

16. Solve the equation $\dfrac{3}{x-2}+1=\dfrac{3}{x-2}$

Multiply both sides of the equation by x - 2 to obtain the equation 3 + (x – 2) = 3.

Note this equation need not be equivalent to the original.

Add -1 to both sides of the equation to obtain x = 2.

Observe that x = 2 creates a zero in the denominator and is therefore not a solution.

Since 2 was the only possible solution, we conclude that the solution set for the original equation is the null set $\varnothing$

17. Solve the equation $\frac{x-4}{x-3}+\frac{x-2}{x-3}=x-3$

Multiply both sides of the equation by x - 3 to obtain the equation

$(x – 4) + (x – 2) = (x - 3)^2$

$2x – 6 = x^2 – 6x + 9$.

Note this equation need not be equivalent to the original.

Add 6 – 2x to both sides of the equation to obtain the equation $x^2 – 8x +15 = 0$

Factor to obtain (x – 3)(x - 5) = 0.

By the Zero Factor Property x – 3 = 0 or x - 5 = 0

From which we obtain the simplest equations x = 3 and x = 5.

Three causes a zero in a denominator and is therefore not a solution whereas 5 does not create a zero in a denominator and is therefore a solution.

We then conclude that the solution set is { 5 }.

18. Solve $I=\frac{E}{R+r}$ **for *r***

Multiply both sides of the equation by R + r to obtain IR + Ir = E.

Add - IR to both sides of the equation to obtain Ir = E – IR

Multiply both sides of the equation by $\frac{1}{I}$ to obtain the desired equation $r=\frac{E-IR}{I}$