

Complex Numbers

Definiton: A **complex number** is a number that can be written in the form $a + bi$ where a and b are real numbers and $i = \sqrt{-1}$.

Definition: Two complex numbers $a + bi$ and $c + di$ are **equal** if and only if $a = c$ and $b = d$.

Definiton: The **real component** of the complex number $a + bi$ is a .

Definiton: The **complex component** of the complex number $a + bi$ is b .

Definiton: The **sum** of two complex numbers $a + bi$ and $c + di$ is defined by

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Definiton: The **opposite** of a complex number $a + bi$ is the complex number $-a - bi$

Definiton: The **difference** $(a + bi) - (c + di)$ is defined in terms of subtraction

Subtraction of Complex Numbers

$$\begin{array}{c}
 (a + bi) - (c + di) \\
 \begin{array}{ccc}
 \downarrow & \leftarrow & \text{Inverse Operation} \\
 \downarrow & \leftarrow & \text{Additive Inverse} \\
 \downarrow & & \\
 (a + bi) + (-c - di) = (a - c) + (b - d)i
 \end{array}
 \end{array}$$

Definiton: The **product** of two complex numbers $a + bi$ and $c + di$ is defined by

$$(a + bi)(c + di) = (ac - bd) + (bc + ad)i$$

Definiton: The **norm** of a complex number $a + bi$ is $a^2 + b^2$

The norm of a complex number is a positive real number

Definiton: The **conjugate** of a complex number $a + bi$ is $a - bi$

Definiton: The **quotient** of a complex number $a + bi$ divided by a real number d is defined by

$$\frac{a + bi}{d} = \frac{a}{d} + \frac{b}{d}i$$

Definiton: The **multiplicative inverse** of a complex number $a + bi$ is $\frac{a - bi}{a^2 + b^2}$

The multiplicative inverse of a complex number is therefore its conjugate divided by its norm.

Definiton: The **quotient** $(a + bi) \div (c + di)$ is defined in terms of multiplication

Division of Complex Numbers

$$(a + bi) \div (c + di)$$

Inverse Operation

Multiplicative Inverse

$$a + bi \cdot \frac{c - di}{c^2 + d^2} = \frac{(a + bi)(c - di)}{c^2 + d^2}$$

Definiton: The **graph** of a complex number $a + bi$ is the point (a, b) .

The distance from the origin to a complex number $a + bi$ is the square root of the norm of the complex number. That is $\sqrt{a^2 + b^2}$