Important Properties of Quadratic Equations and Functions

Definiton: A **quadratic polynomial in one variable** is a polynomial which may be written in the form $ax^2 + bx + c$ where a, b, and c are real numbers and a is not 0.

Definiton: A **quadratic equation in one variable** is an equation which may be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and a is not 0.

Quadratic Formula: Solutions to a quadratic equation in a single variable may be found with the **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note if $b^2 - 4ac < 0$ then $\sqrt{b^2 - 4ac}$ is a complex number

and if $b^2 - 4ac > 0$ then $\sqrt{b^2 - 4ac}$ is a real number

and if $b^2 - 4ac = 0$ then $\sqrt{b^2 - 4ac} = 0$

The Quadratic Foumula will always produce all solutions to a quadratic equation. Those solutions may be real numbers or they may be complex numbers.

Definiton: The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic polynomial $ax^2 + bx + c$. We will also refer to it as the discriminant of the corresponding quadratic equation, or the discriminant of the corresponding quadratic function.

- If the discriminant of a quadratic equation in one variable is positive, the quadratic equation has two real solutions.
  - They represent two x-intercepts of the graph of the corresponding quadratic equation in two variables.
  - They also represent two x-intercepts of the graph of the corresponding quadratic function.

- If the discriminant of a quadratic equation in one variable is zero, the quadratic equation has one real solutions.
  - It represents the single x-intercept of the graph of the corresponding quadratic equation in two variables.
  - It also represents the single x-intercept of the graph of the corresponding quadratic function.

- If the discriminant of a quadratic equation in one variable is negative, the quadratic equations has two complex solutions.
  - They are conjugates of one another.
  - Since only Real Numbers are represented on the real number line, these complex solutions cannot represent x-intercepts of the graph of either the corresponding quadratic equation in two variables or the corresponding quadratic function.

Definiton: A **quadratic equation in two variables** is an equation which may be written in the form $y = ax^2 + bx + c$ where a, b, and c are real numbers and a is not 0.

The graph of a quadratic equation in two variables is a parabola which opens up if $a > 0$ and opens down if $a < 0$.

The y-intercept of a quadratic equation in two variables is (0, c).

The x-intercepts of the graph of a quadratic equation in two variables are found by solving the corresponding quadratic equation in one variable.

Definiton: The **vertex** of a parabola which opens up is the point on the graph with the smallest second coordinate. The vertex of a parabola which opens down is the point on the graph with the largest second coordinate.

The vertex of the graph of a quadratic equation in two variables has first coordinate $\frac{-b}{2a}$.

If the discriminant is positive, the graph has two x-intercepts.
If the discriminant is zero, the graph has one x-intercept and it is the vertex.
If the discriminant is negative, the graph has no x-intercepts.
It is entirely above the x-axis if $a > 0$ (it opens up)
It is entirely below the x-axis if $a < 0$ (it opens down)

Definiton: A **quadratic function** is a function whose rule may be written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and a is not 0.

The graph of a quadratic function is a parabola which opens up if $a > 0$ and opens down if $a < 0$.

The y-intercept of a quadratic function in two variables is (0, c).

Definiton: A zero of a function f is a domain element k such that $f(k) = 0$.

The zeros of a function are found by solving the equation resulting from $f(x) = 0$. In the case of a quadratic function, the zeros are therefore found by solving the corresponding quadratic equation in one variable. This is always possible with the quadratic formula.

The x-intercepts of the graph of a quadratic function are the real zeros of the function.

The vertex of the graph of a quadratic function is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

If the discriminant is positive, the graph has two x-intercepts.
If the discriminant is zero, the graph has one x-intercept and it is the vertex.
If the discriminant is negative, the graph has no x-intercepts.
It is entirely above the x-axis if $a > 0$ (it opens up)
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