

## Important Properties of Quadratic Equations and Functions

**Definiton:** A **quadratic polynomial in one variable** is a polynomial which may be written in the form  $ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers and  $a$  is not 0.

**Definiton:** A **quadratic equation in one variable** is an equation which may be written in the form  $ax^2 + bx + c = 0$  where  $a$ ,  $b$ , and  $c$  are real numbers and  $a$  is not 0.

**Quadratic Formula:** Solutions to a quadratic equation in a single variable may be found with the **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note if  $b^2 - 4ac < 0$  then  $\sqrt{b^2 - 4ac}$  is a complex number

and if  $b^2 - 4ac > 0$  then  $\sqrt{b^2 - 4ac}$  is a real number

and if  $b^2 - 4ac = 0$  then  $\sqrt{b^2 - 4ac} = 0$

The Quadratic Fomula will always produce all solutions to a quadratic equation. Those solutions may be real numbers or they may be complex numbers.

**Definiton:** The expression  $b^2 - 4ac$  is called the **discriminant** of the quadratic polynomial  $ax^2 + bx + c$ . We will also refer to it as the discriminant of the corresponding quadratic equation, or the discriminant of the corresponding quadratic function.

- If the discriminant of a quadratic equation in one variable is positive, the quadratic equation has two real solutions.
  - They represent two x-intercepts of the graph of the corresponding quadratic equation in two variables.
  - They also represent two x-intercepts of the graph of the corresponding quadratic function.
- If the discriminant of a quadratic equation in one variable is zero, the quadratic equation has one real solutions.
  - It represents the single x-intercept of the graph of the corresponding quadratic equation in two variables.
  - It also represents the single x-intercept of the graph of the corresponding quadratic function.
- If the discriminant of a quadratic equation in one variable is negative, the quadratic equations has two complex solutions.
  - They are conjugates of one another.
  - Since only Real Numbers are represented on the real number line, these complex solutions cannot represent x-intercepts of the graph of either the corresponding quadratic equation in two variables or the corresponding quadratic function.

**Definiton:** A **quadratic equation in two variables** is an equation which may be written in the form  $y = ax^2 + bx + c$  where a, b, and c are real numbers and a is not 0.

The graph of a quadratic equation in two variables is a parabola which opens up if  $a > 0$  and opens down if  $a < 0$ .

The y-intercept of a quadratic equation in two variables is (0, c).

The x-intercepts of the graph of a quadratic equation in two variables are found by solving the corresponding quadratic equation in one variable.

**Definiton:** The **vertex** of a parabola which opens up is the point on the graph with the smallest second coordinate. The vertex of a parabola which opens down is the point on the graph with the largest second coordinate.

The vertex of the graph of a quadratic equation in two variables has first coordinate  $\frac{-b}{2a}$ .

If the discriminant is positive, the graph has two x-intercepts.

If the discriminant is zero, the graph has one x-intercept and it is the vertex.

If the discriminant is negative, the graph has no x-intercepts.

It is entirely above the x-axis if  $a > 0$  (it opens up)

It is entirely below the x-axis if  $a < 0$  (it opens down)

**Definiton:** A **quadratic function** is a function whose rule may be written in the form  $f(x) = ax^2 + bx + c$  where a, b, and c are real numbers and a is not 0.

The graph of a quadratic function is a parabola which opens up if  $a > 0$  and opens down if  $a < 0$ .

The y-intercept of a quadratic function in two variables is (0, c).

**Definiton:** A zero of a function f is a domain element k such that  $f(k) = 0$ .

The zeros of a function are found by solving the equation resulting from  $f(x) = 0$ . In the case of a quadratic function, the zeros are therefore found by solving the corresponding quadratic equation in one variable. This is always possible with the quadratic formula.

The x-intercepts of the graph of a quadratic function are the real zeros of the function.

The vertex of the graph of a quadratic function is  $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ .

If the discriminant is positive, the graph has two x-intercepts.

If the discriminant is zero, the graph has one x-intercept and it is the vertex.

If the discriminant is negative, the graph has no x-intercepts.

It is entirely above the x-axis if  $a > 0$  (it opens up)

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